

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-III(OLD) EXAMINATION – SUMMER 2023****Subject Code:130001****Date:21-07-2023****Subject Name:Mathematics-III****Time:02:30 PM TO 05:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) (i) Solve $\frac{dy}{dx} + y \sin x = e^{\cos x}$ **03**

(ii) Solve $y e^x dx + (2y + e^x) dy = 0$ **04**

(b) Find the power series solution of differential equation $\frac{d^2y}{dx^2} + xy = 0$ **07**

Q.2 (a) Solve $y'' - 3y' + 2y = e^{3x}$ **07**

(b) Using the method of variation of parameter, solve $y'' + 9y = \sec 3x$ **07**

OR

(b) Solve $(D^2 - 1)y = x e^x$ where $D = \frac{d}{dx}$ **07**

Q.3 (a) Using the method of undetermined coefficient, solve **07**

$$y'' - 2y' + 5y = 5x^3 - 6x^2 + 6x$$

(b) Find the Fourier cosine series for $f(x) = \pi - x$, $0 < x < \pi$ **07**

OR

Q.3 (a) Solve $(D^3 - D)y = x^3$ **07**

(b) Find the Fourier series of $f(x) = x$ in $-\pi < x < \pi$ **07**

Q.4 (a) State Convolution theorem and by using it to find the Inverse Laplace transform **07**
of $\frac{s}{(s^2 + a^2)^2}$

(b) (i) Find $L\left(\frac{\sin 2t}{t}\right)$ **03**

(ii) Find $L[e^{-2t}(\sin 4t + t^2)]$ **04**

PTO

OR

Q.4 (a) Prove that $L(1) = \frac{1}{s}$ and $L(\sinh at) = \frac{a}{s^2 - a^2}$ **07**

(b) Solve $y'' + y = \sin 2t$, $y(0) = 2$, $y'(0) = 1$ by using Laplace Transform. **07**

Q.5 (a) (i) Define: (1) Beta function (2) Gamma function **03**

(ii) Form the partial differential equation of $f(x + y + z, x^2 + y^2 + z^2) = 0$ **04**

(b) Using method of separation of variables, solve **07**

$$\frac{\partial u}{\partial x} = 2 \frac{\partial u}{\partial t} + u, \quad u(x, 0) = 6e^{-3x}$$

OR

Q.5 (a) Solve (i) $z = px + qy + p^2q^2$ **03**

(ii) $p^2 - q^2 = x - y$ **04**

(b) Solve $x^2p + y^2q = z^2$ **07**