

GUJARAT TECHNOLOGICAL UNIVERSITY
BE - SEMESTER– IV(OLD) EXAMINATION – SUMMER 2023

Subject Code:140001**Date:07-07-2023****Subject Name:Mathematics-IV****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1 (a)** For $z_1 = 5 + 3i$ and $z_2 = 2 + 4i$, find $\text{Re}(Z_1)$ and $\text{Im}(Z_1)$ using the formula. Also, 07
 verify that $\overline{z_1 z_2} = \overline{z_1} \overline{z_2}$.
- (b)** Solve $z^4 + 1 = 0$ and locate all roots in the Argand diagram. 07

- Q.2 (a)** Find and sketch the image of region $0 < y < 1/2$ under the transformation $w = \frac{1}{z}$. 07
- (b)** Evaluate $\int_0^{2+i} z^2 dz$ along the line $y = \frac{x}{2}$. 07

OR

- (b)** Determine analytic function whose real part is $u(x, y) = x^2 - y^2$ 07
- Q.3 (a)** Find the bilinear transformation that maps, respectively points $i, 1, -i$ in z -plane 07
 onto the points $-i, 1, i$ in w -plane.
- (b)** Evaluate $\int_C \frac{\cos \pi z^2}{(z-2)(z-1)} dz$ counter clockwise around the circle $C: |z| = 3$ 07

OR

- Q.3 (a)** Evaluate $\int_0^\infty \frac{dx}{(a^2 + x^2)^2}$ 07
- (b)** Using Newton's forward difference method, find the approximate value of (1.3) 07
 from the following data.

x	1	2	3	4
f(x)	1.1	4.2	9.3	16.4

- Q.4 (a)** Using Newton's divided difference method, find the approximate value of (10.5) 07
 from the following data.
- | | | | | |
|-------------|---------------|---------------|---------------|---------------|
| x | 10 | 11 | 13 | 17 |
| f(x) | 2.3026 | 2.3979 | 2.5649 | 2.8332 |
- (b)** Evaluate $\int_0^1 \exp(-x^2) dx$ by using the Gaussian integration formula for $n=3$. 07

OR

- Q.4 (a)** Solve the following system of equations using Gauss elimination method. 07
 $6x + 2y + 8z = 26, \quad 8y + 2z = -7, \quad 3x + 5y + 2z = 8$

- (b) Solve the following system of equations using Gauss – Seidel method correct to four decimal places. **07**

$$30x - 2y + 3z = 75, \quad 2x + 2y + 18z = 30, \quad x + 17y - 2z = 48$$

- Q.5 (a)** Find a positive root of the equation $x - \cos x = 0$ **07**

- (b) Using Newton's- Raphson method, find a real root of the equation **07**
 $x^3 - 3x - 5 = 0$

OR

- Q.5 (a)** Using Euler's method, find $y(0.2)$ given that $\frac{dy}{dx} = y - \frac{2x}{y}$, $y(0) = 1$, $h = 0.1$ **07**

- (b) Apply Runge-Kutta method of fourth order to calculate $y(0.2)$ given that **07**
 $\frac{dy}{dx} = x + y$, $y(0) = 1$, $h = 0.1$
