

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER- IV(NEW) EXAMINATION – SUMMER 2023****Subject Code:3142008****Date:11-07-2023****Subject Name:Numerical Methods****Time:10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1** (a) Give the definition of Error and Explain three Main Types of it with formulas. **03**
- (b) Use the Regula-Falsi(False Position) Method to find a root of the equation $x^3 - 4x + 1 = 0$ correct up to three decimal places if the root lies between 0 and 1 **04**
- (c) Fit a Second Degree Parabola $y = a + bx + cx^2$ to the following data using Method of Least Squares Also find $y(6)$ **07**

x	1	2	3	4	5
y	5	12	26	60	97

- Q.2** (a) Using Trapezoidal rule find the value of the integral $\int_0^2 \frac{x}{\sqrt{2+x^2}} dx$ **03**
Dividing the interval into four equal parts.
- (b) Using the Bisection Method find a root of $f(x) = \cos x - xe^x = 0$ in Four Steps. **04**
- (c) Use Gauss Seidel Method to Solve the following equations **07**
 $8x + y + z = 5, x + 8y + z = 5, x + y + 8z = 5$

OR

- (c) Determine $y(12)$ using Lagrange's Interpolation method using the following tabulated data. **07**

x	11	13	14	18	20	23
y	25	47	68	82	102	124

- Q.3** (a) Using Secant Method find the Positive solution of $f(x) = x - 2\sin x = 0$ correct up to three decimal places starting from $x_0 = 2$ and $x_1 = 1.9$ **03**

- (b) If $\Delta x = 0.005$ and $\Delta y = 0.001$ be the absolute errors in $x = 2.11$ and $y = 4.15$ then find the Relative Error in the computation of $x + y$ 04
- (c) From the following table find the value of $f(6)$ using Newton's Divided Difference formula 07

x	1	2	7	8
$f(x)$	1	5	5	4

OR

- Q.3 (a) Prove that (i) $(1 + \Delta)(1 - \nabla) = 1$ 03
(ii) $\nabla = \Delta E^{-1} = E^{-1} \Delta = 1 - E^{-1}$
- (b) Find an Iterative Formula for $\sqrt{N}$, where N is a Positive number and hence Find $\sqrt{28}$ correct up to Three decimal places Using Newton-Raphson Method. 04
- (c) Obtain Cubic Spline for every Subinterval and evaluate $y(1.5)$ for the following data: 07

x	1	2	3	4
y	1	2	5	11

- Q.4 (a) Write the formula for Runge-Kutta Fourth Order Method. 03
- (b) Using Euler's Method find the Approximate solution of $\frac{dy}{dx} = x + y$ with $y(0) = 1$ when $x = 0.1$ and 0.2 with steps size $h = 0.05$ 04
- (c) Given $\frac{dy}{dx} = \frac{1}{2}(1 + x^2)y^2$ and $y(0) = 1, y(0.1) = 1.06, y(0.2) = 1.12, y(0.3) = 1.21$ Evaluate $y(0.4)$ using Milne's Predictor and Corrector Method. 07

OR

- Q.4 (a) Evaluate the Integral $\int_0^6 \frac{1}{1+x} dx$ taking $h = 1$ by Simpson's $\frac{1}{3}$ Rule. 03
- (b) Fit a Straight line $y = a + bx$ to the following Data: 04

x	1	2	3	4	6	8
y	2.4	3	3.6	4	5	6

- (c) Use Modified Euler's Method to Solve $\frac{dy}{dx} = x + 3y$ subject to $y(0) = 1$ and Hence find an approximate value of $y(1)$ with $h = 0.1$ 07

Q.5 (a) By Simpson's $\frac{3}{8}$ rule evaluate $\int_0^1 \frac{\sin x}{x} dx$ taking $h = \frac{1}{6}$ **03**

(b) Find Inverse of the Matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$, Using Gauss Jordan Method. **04**

(c) Determine the Polynomial by Newton's forward difference Interpolation formula from the following table: **07**

x	0	1	2	3	4	5
y	-10	-8	-8	-4	10	40

OR

Q.5 (a) If $\pi = \frac{22}{7}$ is approximated as 3.14. Find the Absolute error, Relative error and Percentage error. **03**

(b) Fit a curve $y = ae^{bx}$ to the following data using Method of Least Square **04**

x	1	3	5	7	9
y	115	105	95	85	80

(c) Solve the following system of equations using Gauss Elimination Method $2x + y + z = 10, 3x + 2y + 3z = 18, x + 4y + 9z = 16$ **07**
