

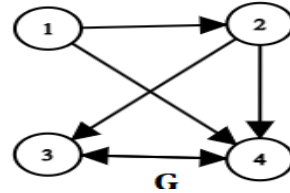
GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER– IV(NEW) EXAMINATION – SUMMER 2023****Subject Code:3144202****Date:27-07-2023****Subject Name:Discrete Mathematics and Graph Theory****Time:10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1** (a) Prove that: $(A \cap B) \cup (A - B) = A$ and $A \cup (B - A) = A \cup B$ **03**
- (b) Let G be a graph with n vertices and m edges such that vertices have degree k or $k+1$. Prove that if G has N_k vertices of degree k and N_{k+1} vertices of degree $k+1$ then $N_k = (k+1)n - 2m$ **04**
- (c) (I) Define weakly connected graph, Boolean matrix and Euler path. **03**
 (II) For two sets A and B show that $f: A \times B \rightarrow B \times A$ such that $f(a, b) = (b, a)$ is a bijective function. **04**
- Q.2** (a) In a survey of 600 students in a school, 150 students were found to be drinking tea and 225 drinking coffee, 100 were drinking both tea and coffee. Find how many students drinking neither tea nor coffee. **03**
- (b) If R be the relation defined in Q - the set of rational numbers by $R = \left\{ \left(\frac{a}{b}, \frac{c}{d} \right) \in Q \times Q : ad = bc \right\}$, then show that R is an equivalence relation. **04**
- (c) Solve the recurrence relation $S_n - 4S_{n-1} + 3S_{n-2} = 5^n$ **07**
- OR**
- (c) Solve the recurrence relation $S_n - 8S_{n-1} - 12S_{n-2} = 0$, $S_0 = 54$ and $S_1 = 308$ **07**
- Q.3** (a) Define complete graph and find order and size of the graph K_{2023} **03**
- (b) Verify that the proposition $(p \rightarrow q) \leftrightarrow (q \vee \neg p)$ is tautology. **04**
- (c) Show that $(R, +, \times)$ is an integral domain, where $R = \{a + b\sqrt{7} \mid a, b \in I\}$ **07**
- OR**
- Q.3** (a) Draw Hasse diagram for $P = \{1, 2, 3, 4, 6, 12\}$ and $\leq$ is a relation such that $x \leq y \Leftrightarrow x \mid y$. **03**
- (b) Check whether the relation R defined by $R = \{(a, b) : a \leq b^3\}$ is reflexive, symmetric or transitive. **04**
- (c) (I) Does there exist a graph with 20 edges and each vertex of degree 3? **03**
 (II) If $f(x) = \frac{1}{1-x}$ then show that $f(f(f(x))) = x$. **04**
- Q.4** (a) Prove that ${}^nC_1 + 6({}^nC_2) + 6({}^nC_3) = n^3$ **03**
- (b) For a group G , show that the mapping $f: G \rightarrow G$ defined by $f(x) = x^2$ is a homomorphism if and only if G is abelian. **04**
- (c) Prove that following argument is valid: **07**
 No mathematician is a fool. No one who is not a fool is an administrator.
 Sita is a mathematician. Therefore, Sita is not an administrator.

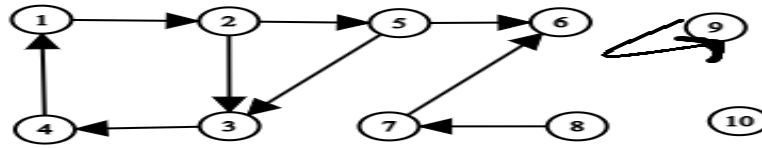
OR

- Q.4 (a)** Determine whether the given digraph is strongly, unilaterally and weakly connected.



03

- (b)** Find a Node base for the following graph:



04

- (c)** Show that arbitrary intersection of subgroups of a group is a subgroup of a group. Will union of two subgroups be also subgroup of group?

07

- Q.5 (a)** Show that the symmetric group S_n for $n \geq 3$ is not abelian.

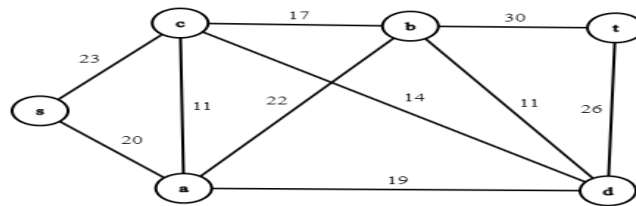
03

- (b)** If S_{42} is the set of all divisor of 42 and D is the relation “divisor” on S_{42} , prove that $\{S_{42}, D\}$ is complemented Lattice.

04

- (c)** Find Shortest path between s and t in graph shown below using Dijkstra’s Algorithm:

07



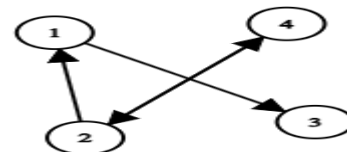
OR

- Q.5 (a)** Show that $H = \left\{ \begin{bmatrix} 1 & b \\ 0 & 1 \end{bmatrix} : b \in R \right\}$ is a subgroup of G , where G is the group of all non-singular upper triangular matrices of order 2×2 over R with usual matrix multiplication.

03

- (b)** Using Warshall’s algorithm find path matrix of the given digraph.

04



- (c)** Define Isomorphic graphs. Check whether the following graphs are isomorphic?

07

