

Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-III (OLD) EXAMINATION – SUMMER 2024

Subject Code:130001

Date:11-07-2024

Subject Name:Mathematics-III

Time:10:30 AM TO 01:30 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1** (a) (i) Solve $y' + y \sin x = e^{\cos x}$ 03
(ii) Solve $(x^2 - 2xy - y^2)dx - (x + y)^2 dy = 0$ 04
- (b) Find the power series solution of the equation $\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = 0$. 07
- Q.2** (a) (i) Solve $y'' + 9y = \cos 4x$ 03
(ii) Using the method of variation of parameter, solve $y'' - 3y' + 2y = e^x$ 04
- (b) Using the method of undetermined coefficient, solve $y'' - 2y' = e^x \sin x$ 07
- OR**
- (b) Solve the equation by series method $4xy'' + 2y' + y = 0$ 07
- Q.3** (a) Find the Fourier series of $f(x) = x^2$ in the interval $(-\pi, \pi)$. 07
(b) Find half range Fourier sine series for the given function $f(x) = x$ in $0 < x < 2$. 07
- OR**
- Q.3** (a) Find the Fourier series of $f(x) = |x|$, $-\pi < x < \pi$. 07
(b) Find the Half range Fourier cosine series of $f(x) = x \sin x$, $0 \leq x \leq \pi$. $f(x + 2\pi) = f(x)$ 07
- Q.4** (a) (1) Find the Laplace transform of the function $f(t) = e^t \sin t$. 03
(2) Find the inverse Laplace transform of the function $F(s) = \frac{1}{s^2 + 8s + 16}$. 04
- (b) Solve the differential equation using Laplace Transformation method $(D^2 - 3D + 2)y = 1 - e^{-2t}$, Given that $y(0) = 1$, $y'(0) = 0$, $t > 0$. 07
- OR**
- Q.4** (a) (1) Find the Laplace transform of the function $f(t) = e^{-2t}(t^2 + \sin 4t)$ 03
(2) Find the inverse Laplace transform of the function $F(s) = \frac{6s - 4}{s^2 - 4s + 20}$ 04
- (b) Define Convolution theorem for Laplace transform. Using Convolution theorem to find Laplace inverse of the function $F(s) = \frac{1}{s^2(s^2 - a^2)}$ 07
- Q.5** (a) (1) Form the partial differential equation of $z = axy + b$. 03
(2) Prove that $\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$ 04

- (b) Solve by the method of separation of variables $\frac{\partial^2 u}{\partial x^2} = \frac{1}{2} \frac{\partial u}{\partial t}$ over $0 < x < 3, t > 0$. **07**
 for the boundary conditions $u(0, t) = u(3, t) = 0$ and initial condition $u(x, 0) = 5 \sin 4\pi x$.

OR

- Q.5 (a) (1)** Solve the partial differential equation $pq = xy$ **03**

- (2) Show that $\int_0^1 x^2 (1-x)^3 dx = \frac{1}{60}$. **04**

- (b) Using Charpit's method to solve the $z^2 = pqxy$ **07**
