

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV (NEW) EXAMINATION – SUMMER 2024****Subject Code:3142008****Date:18-07-2024****Subject Name:Numerical Methods****Time:10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Marks

Q.1 (a) Using Simpson's 1/3 rule, evaluate $\int_{-2}^6 (1+x^2)^{3/2} dx$ by taking 8 subintervals. **03**

(b) The population of the town is given below. Estimate the population for the year 1930 using Newton's backward interpolation. **04**

Year	1891	1901	1911	1921	1931
Population (in thousand)	46	66	81	93	101

(c) Solve the following equations by Gauss-Seidel method: **07**
 $20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25$

Q.2 (a) Prove that $\Delta \nabla = \nabla \Delta = \Delta - \nabla$. **03**

(b) Find a real root of the equation $x^3 + x^2 - 1 = 0$ using Method of fixed iterations, correct up to 3 decimal places. **04**

(c) Find the positive real root of equation $x = e^{-x}$ using Newton-Raphson method, correct up to 4 decimal places. **07**

OR

(c) Find root of an equation $xe^x = \cos x$ using False Position method correct to 4 significant digits. **07**

Q.3 (a) Find the dominant Eigen value of $A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$ by using power method and the corresponding Eigen vector. **03**

(b) Using fourth order Runge-Kutta method, find $y(0.2)$ with $h = 0.1$ given that **04**

$$10 \frac{dy}{dx} = x^2 + y^2, y(0) = 1.$$

(c) Determine the value of y using improved Euler's method when $x = 0.1$ given that $y(0) = 1$ and $y' = x^2 + y$. Choose the step size $h = 0.05$. **07**

OR

Q.3 (a) Determine the interpolating polynomial of degree three using Langrange's interpolation formula. **03**

x	0	1	3	4
$y = f(x)$	-12	0	12	24

(b) Use Euler's methods to obtain an approximate value of $y(1)$ for the equation **04**

$$\frac{dy}{dx} = x + y, y(0) = 1.$$

(c) Determine the solution of $y'' = x + y$ with $y(0) = 0, y(1) = 0$, Using $h = 0.5$ implement finite difference approach. **07**

- Q.4 (a)** Evaluate $\int_0^1 (1 + e^{2x} \sin 4x) dx$ by using Trapezoidal rule taking $h = 0.25$. **03**

- (b)** Given that **04**

x	1	1.05	1.1	1.15	1.2	1.25	1.3
y	1	1.0247	1.0488	1.0723	1.0954	1.1180	1.1401

Find $\frac{dy}{dx}$ at $x = 1.25$.

- (c)** Solve the following systems by Gauss-Jordan method. **07**

$$x + 2y + z = 8, 2x + 3y + 4z = 20, 4x + 3y + 2z = 16$$

OR

- Q.4 (a)** Compute $\int_{-1}^1 \frac{1}{x^2+1} dx$ using Gaussian formula for $n = 3$. **03**

- (b)** Given that **04**

x	0	1	2	3	4
y	4	8	15	7	6

Find $\frac{d^2y}{dx^2}$ at $x = 0$.

- (c)** Solve the following system using matrix inversion method. **07**

$$x + 2y + 3z = 1, 2x + 5y + 3z = 1, x + 8z = 2$$

- Q.5 (a)** Using Newton divided difference formula find $f(9.2)$ from the following table: **03**

x	8.0	9.0	9.5	11.0
$y = f(x)$	2.079442	2.197225	2.251292	2.397895

- (b)** Define: true percentage relative error, approximate percentage relative error. **04**

- (c)** Fit a second degree parabola to the following data: **07**

x	1	1.5	2	2.5	3	3.5	4
y	1.1	1.3	1.6	2.0	2.7	3.4	4.1

OR

- Q.5 (a)** Fit a straight line to the following data: **03**

x	1	2	3	4	6	8
y	2.4	3	3.6	4	5	6

- (b)** Find the absolute error and relative error in $\sqrt{3} + \sqrt{5} + \sqrt{7}$ correct to three significant digits. **04**

- (c)** Find the cubic spline in the interval $[1, 2]$ and evaluate $y(1.5)$ for the following data: **07**

x	1	2	3	4
y	1	2	5	11
