

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV (NEW) EXAMINATION – SUMMER 2024****Subject Code:3144007****Date:03-07-2024****Subject Name: Numerical and Statistical Methods****Time:10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.
5. Standard Normal Table is to be provided as and when necessary.

Q.1 (a) A shop sells 10 tube lights out of which 3 are defective. A person buys four tube lights. Find the chance that the tube lights that he buys work. **03**

(b) Find the median: **04**

Marks	0-10	10-20	20-30	30-40	40-50
No. of Students	7	25	30	10	6

(c) Evaluate $f(3)$ using Newton's divided difference interpolation formula: **07**

x	0	1	2	4	5	6
f(x)	1	14	15	5	6	19

Q.2 (a) A fair coin is tossed, and a fair die is thrown. Let A be the event that a head is tossed, and B be the event that an odd number is thrown. Calculate $P(A \cap B)$ and $P(A \cup B)$. **03**

(b) Given $f(2) = 9$, and $f(6) = 17$. Find an approximate value for $f(5)$ by the method of Lagrange's interpolation. **04**

(c) Evaluate the two lines of regression from the following results: **07**
 $\bar{x} = 90, \bar{y} = 70, n = 10, \sum x^2 = 6360, \sum y^2 = 2860, \sum xy = 3900$

OR

(c) The heights of students can be modelled with normal distribution with mean 168 cm and standard deviation 4.5 cm. Find the probability that the height of a randomly selected student will be **07**

- (i) Less than 172.5 cm
- (ii) Between 159 cm and 163.5 cm.

Q.3 (a) Obtain the standard deviation of 4, 8, 10, 12 and 16. **03**

(b) Use 2-point Gaussian quadrature formula to approximate $\int_0^1 e^{-x^2} dx$. **04**

(c) A diagnostic test to detect disease X has 99% accuracy and 60% of all people have the disease X. If a patient tests positive, what is the probability that the patient actually has the disease? **07**

OR

Q.3 (a) Use bisection method to determine the approximate root of $x^3 + 4x^2 - 10 = 0$ corrected up to two decimal points. **03**

(b) Mass-produced needles are packed in boxes of 1000. It is believed that on average, 1 needle in 2000 is defective. Using Poisson process, calculate the probability that a box contains 2 or more defectives. **04**

- (c) Fit an exponential curve $y = ae^{bx}$ from the data: 07

x	0	0.5	1	1.5	2	2.5
y	0.1	0.45	2.15	9.15	40.35	180.75

- Q.4 (a) Approximate the value of y (1) using Euler's method when $\frac{dy}{dx} = xy + 1$ and y (0) = 0 using h = 0.5. 03

- (b) Let X is a discrete random variable with probability distribution 04

X	1	4	6
P (x = X)	0.2	0.55	0.25

Evaluate the mean and the standard deviation of the distribution.

- (c) Using fourth order Runge-Kutta method, evaluate y (0.2) from the differential equation $\frac{dy}{dx} = -2x - y$, where y (0) = - 1 and h = 0.1. 07

OR

- Q.4 (a) A teacher claims that the mean score of students in his class is greater than 82 with a standard deviation of 20. If a sample of 81 students was selected with a mean score of 90 then check if there is enough evidence to support this claim at a 0.05 significance level. (Given $z_{0.05} = 1.645$) 03

- (b) The ranks of 9 students in history and algebra are as follows: 04

Hist.	3	5	1	6	7	2	8	9	4
Alg.	5	3	2	6	8	1	7	9	4

Calculate Spearman's rank correlation coefficient.

- (c) Use the Newton forward interpolation formula to find f (2.4): 07

x	2	4	6	8	10
f (x)	9.68	10.96	12.32	13.76	15.28

- Q.5 (a) An unbiased coin is tossed eight times. Calculate the probability of less than 4 heads by binomial distribution. 03

- (b) Apply Simpson's $\frac{3}{8}$ th rule to calculate $\int_0^1 \frac{dx}{1+x^2}$ by taking n = 5. 04

- (c) A group of 5 patients was treated with medicine A and have the weights 42, 39, 38, 60 & 41 kgs. A second group of 7 patients from the same hospital was treated with medicine B and have weights 38, 42, 56, 64, 68, 69 & 62 kgs. Find whether there is any difference in variability between the medicines at 5% significant level. (Given $F_{0.05} = 6.16$) 07

OR

- Q.5 (a) A continuous random variable X has density function $f(x) = k$, $0 \leq x \leq 1$. Determine the constant k and find $P(X \leq \frac{1}{4})$. 03

- (b) Find an approximate root of the equation $x^3 - x - 1 = 0$ using secant method corrected up to two decimal places. 04

- (c) A genetic engineer was attempting to cross a tiger and a cheetah. She predicted the traits to be in the following ratio – 4 stripes only: 3 spots only: 9 both stripes and spots. After the cross was performed, it was observed that 50 were with stripes only, 41 were with spots only and 85 were with both. According to the Chi-square test at 5% level, did she get the predicted outcome? (Given that $\chi^2_{0.05} = 5.991$) 07
