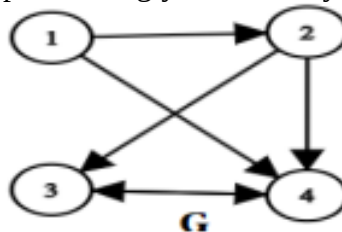


GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV (NEW) EXAMINATION – SUMMER 2024****Subject Code:3144202****Date:10-07-2024****Subject Name: Discrete Mathematics and Graph Theory****Time:10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

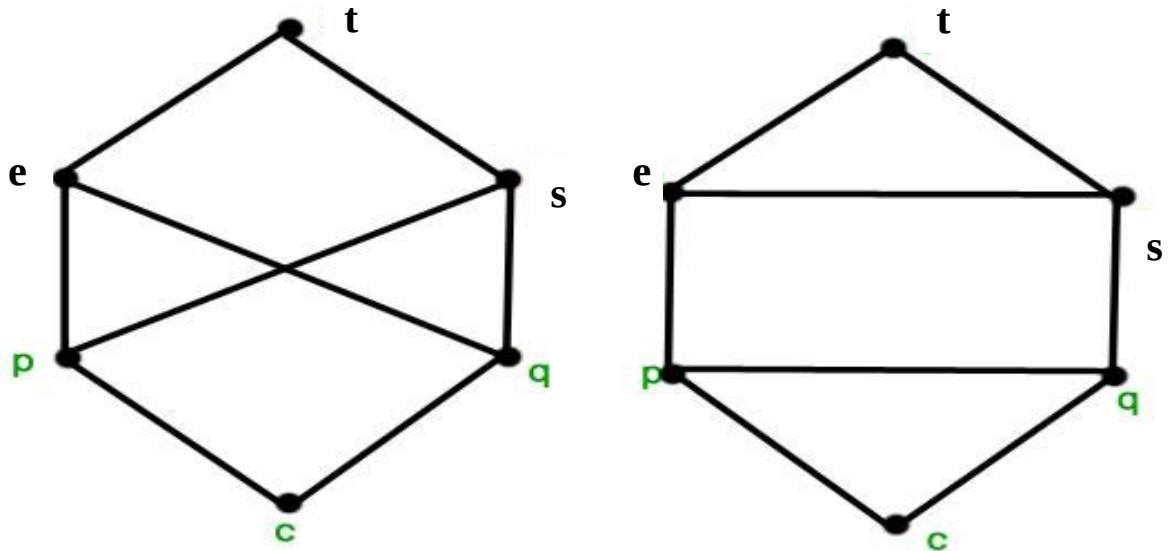
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- | | MARKS |
|--|-----------|
| Q.1 (a) For $A = \{2,3,4,5\}$, $B = \{3,4,5,6\}$, $C = \{4,5,7,8\}$ find
(i) $(A \cup B) \cap (A \cup C)$
(ii) $(A \cap B) \cup (A \cap C)$ | 03 |
| (b) Let $f: R \rightarrow R$, $f(p) = 5p + 4$ and $g: R \rightarrow R$, $g(p) = 5p + m$ If $f \circ g = g \circ f$ then find m. | 04 |
| (c) (i) If p stands for 'it is Hot' and q stands for 'it is raining, translate the following into symbols
(a) It is cold or it is raining
(b) It is Hot and it is raining
(c) A necessary and sufficient condition for raining is that it be Hot

(ii) For two sets A and B show that $f: A \times B \rightarrow B \times A$ such that $f(a, b) = (b, a)$ is a bijective function. | 03 |
| | 04 |
| Q.2 (a) Suppose 8 students are staying in a hall in a hostel and they are allowed 8 beds. Among them, Lalit does not want to bed next to Dinesh because Dinesh snores. Then in how many ways can you allot the beds? | 03 |
| (b) Define Lattice. Determine whether POSET $\{\{1, 2, 3, 4, 5\}; \}$ is a lattice. | 04 |
| (c) Define Equivalence relation. Given $S = \{1,2,3,\dots,12\}$ and $R = \{\{x, y\} x - y \text{ is divisible by } 3\}$. Show that R is an equivalence relation on. | 07 |
| OR | |
| (c) Check whether $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ is Tautology, contradiction or contingency. | 07 |
| Q.3 (a) Draw the Hasse diagram for the poset $(\{2, 4, 5, 10, 12, 20, 25\},)$. Hence, find there maximal and minimal elements. | 03 |
| (b) Solve the recurrence relation $a_n - 3a_{n-1} = 2$; $n \geq 1$ with $a_0 = 1$ using the method of undetermined coefficients. | 04 |
| (c) Show that $(Z_4, +_4)$ is a cyclic group and find all the subgroups of $(Z_4, +_4)$. | 07 |
| OR | |
| Q.3 (a) Determine whether the given digraph is strongly, unilaterally and weakly connected. | 03 |



- (b) Draw Hasse Diagram for the lattice (S_{30}, D) where S_{30} is the set of divisors of 30 and D is the relation divides **04**
- (c) Use generating function to solve a recurrence relation $a_n - 5a_{n-1} + 6a_{n-2} = 3^n, n \geq 2; a_0 = 0, a_1 = 2$. **07**
- Q.4** (a) Define Cyclic Group and abelian group **03**
- (b) Define Tree, Path, Subgraph and Null Graph **04**
- (c) Let G be the set of non-zero real numbers. Define binary operation $(*)$ on G as $a*b = \frac{ab}{2}$. Show that an algebraic structure $(G, *)$ is an abelian group. **07**
- OR**
- Q.4** (a) Define Ring **03**
- (b) Define complement of a sub graph by drawing the graphs. **04**
- (c) Let $G = \{(a, b) | a, b \in R\}$. Define binary operation $(*)$ on G as $(a, b), (c, d) \in G$, such that $(a, b) * (c, d) = (ac, bc + d)$. Show that an algebraic structure $(G, *)$ is a group. **07**
- Q.5** (a) Define Bipartite Graph with Example **03**
- (b) Determine the number of edges in a graph with 6 nodes, 2 of degree 4 and 4 of degree 2. Draw two such graphs **04**
- (c) Define Isomorphic graphs. Check whether the following graphs are isomorphic? **07**



OR

- Q.5** (a) Define Complete Graph and Regular graph **03**
(b) A graph G has 15 edges, 3 vertices of degree 4 and other vertices of degree 3. Find the number of vertices in G **04**
(c) Find the shortest path between each pair of vertices for a simple using Warshall's Algorithm digraph. **07**

