

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV EXAMINATION – SUMMER 2025****Subject Code:140001****Date:31-05-2025****Subject Name: Mathematics-IV****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1** (a) Show that the function $u = x^2 - y^2$ is harmonic and find the corresponding analytic function $f(z) = u + iv$. **07**
- (b) Find and plot all the roots of $\sqrt[3]{8i}$ **07**
- Q.2** (a) Define mobius transformation and determine the mobius transformation that maps the points $z_1 = 0, z_2 = 1, z_3 = \infty$ onto the points $w_1 = -1, w_2 = -i, w_3 = 1$. **07**
- (b) Evaluate $\oint_c \frac{e^{2z}}{(z+1)^4} dz$, where c is the circle $|z| = 2$ **07**
- OR**
- (b) If $u(x, y)$ is a harmonic function then prove that $f(z) = u_x - iu_y$ is an analytic function. **07**
- Q.3** (a) Evaluate $\int_c (x^2 + ixy) dz$ from $(1,1)$ to $(2,4)$ along the curve $x = t, y = t^2$. **07**
- (b) Expand $f(z) = \frac{1}{(z+1)(z-2)}$ in Laurent series in the region **07**
- (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$.
- OR**
- Q.3** (a) Evaluate by using Cauchy's residue theorem $\oint_c \frac{5z-2}{z(z-1)} dz$, where c is $|z| = 2$. **07**
- (b) Show that $\int_0^{2\pi} \frac{d\theta}{\cos\theta+2} = \frac{2\pi}{\sqrt{3}}$. **07**
- Q.4** (a) Use bisection method to find the root of the equation $x^3 - 4x - 9 = 0$ correct to two decimal places. **07**
- (b) State Simpson's $\frac{1}{3}$ rule and using it evaluate $\int_{-2}^6 (1+x^2)^{\frac{3}{2}} dx$ by taking 6 subintervals. **07**
- OR**
- Q.4** (a) Use Euler's method to find $y(1)$ from the differential equation $\frac{dy}{dx} = x + y, y(0) = 1$. (Taking $h = 0.1$) **07**
- (b) Using Newton's divided difference formula, find the polynomial of the lowest possible degree which takes the values 3,12,15,-29 when x has the values 3,2,1,-1. **07**

- Q.5 (a)** Find a real root of the equation $x^3 - 5x + 3 = 0$ correct to three decimal places using Newton-Raphson method. **07**
- (b)** Apply Gauss-Seidel method to solve the system of linear equations. **07**
- $$\begin{aligned} 3x - 0.1y - 0.2z &= 7.85 \\ 0.1x + 7y - 0.3z &= -19.3 \\ 0.3x - 0.2y + 10z &= 71.4 \end{aligned}$$
- OR**
- Q.5 (a)** Apply fourth order Runge-Kutta method to determine $y(0.1)$. (Taking $h = 0.1$) **07**
- $$\frac{dy}{dx} = 2x + y, \quad y(0) = 1$$
- (b)** Estimate $f(1.3)$ using Newton's forward formula. Given that $f(1) = 1.1, f(2) = 4.2, f(3) = 9.3, f(4) = 16.4$. **07**
