

Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Bachelor of Engineering - SEMESTER - IV EXAMINATION - SUMMER 2025

Subject Code: 3144202

Date: 27-05-2025

Subject Name: Discrete Mathematics and Graph Theory

Time: 10:30 AM TO 01:00 PM

Total Marks: 70

Instructions

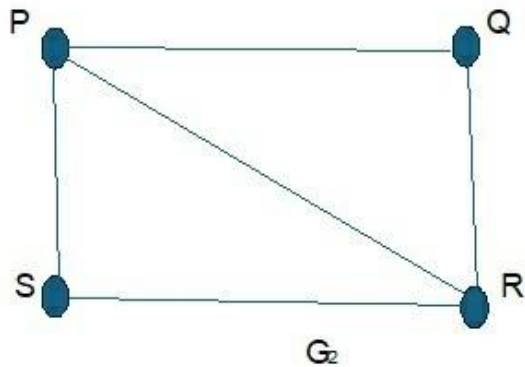
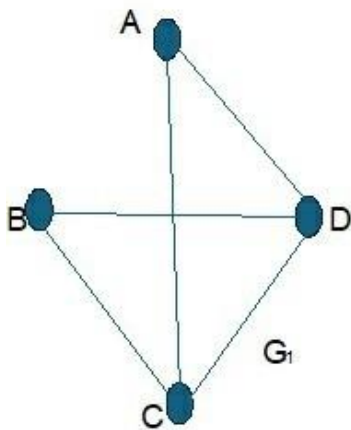
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.

	Marks
Q.1 (a) Define Power set of a set and write the power set for $A=\{1,2,3,4\}$.	03
(b) Define one one and onto function. Find the inverse of $f:R \rightarrow R, f(x) = 7x - 5$, if possible.	04
(c) (i) In a course, a professor gives five grades $\{A, B, C, D, F\}$ to his students in his class. What is the minimum number of students required so that four of them are guaranteed to get the same grade ? (ii) How many integers are there between 99 and 1000 having at least one of their digits 7?	07
Q.2 (a) Define Bounded lattice and Complemented lattice with one example for both.	03
(b) Check whether $p \rightarrow (q \vee r)$ and $(p \rightarrow q) \vee (p \rightarrow r)$ are logically equivalent or not.	04
(c) Define Tautology, Contradiction and Contingency for statements. Check whether $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ is tautology, contradiction or contingency.	07
OR	
(c) Use generating function to solve a recurrence relation $a_n = 3 a_{n-1} + 2, n \geq 1$, with $a_0 = 1$.	07
Q.3 (a) Define (i) Domain and Range of a relation, (ii) Void relation and (iii) Universal relation.	03
(b) Let the given set $X = \{1, 2, 3, 4\}$ and the relation $R = \{(x, y)/ x > y\}$. Write the relation R, draw the corresponding graph and also write its relation matrix.	04
(c) Define equivalence relation. Show that the relation $R = \{(x, y)/ x - y \text{ is divisible by } 3\}$ on set of integer Z is equivalence relation.	07
OR	
(a) Define Partially Ordered Set. Draw Hasse diagram for the given set $X = \{2, 3, 6, 12, 24, 36\}$.	03
(b) Define Lattice. Draw Hasse diagram for lattice (S_{45}, D) , where S_{45} is the divisors of 45 and D is the relation divides.	04
(c) Define (i) Connected graph, (ii) Strongly connected graph, (iii) Weakly connected graph and (iv) Unilaterally connected graph with one example for each of them.	07

- Q.4 (a)** Define Subgroup of a group. Check whether $(3\mathbb{Z}, +)$ is a subgroup of $(\mathbb{Z}, +)$ or not. 03
- (b)** Define semigroup and monoid. Give one example of a semigroup which is not a monoid. 04
- (c)** For a set G of nonzero real numbers, a binary operator $*$ is defined as $a*b = ab/4$. 07
Prove that an algebraic structure $(G, *)$ is an abelian group.

OR

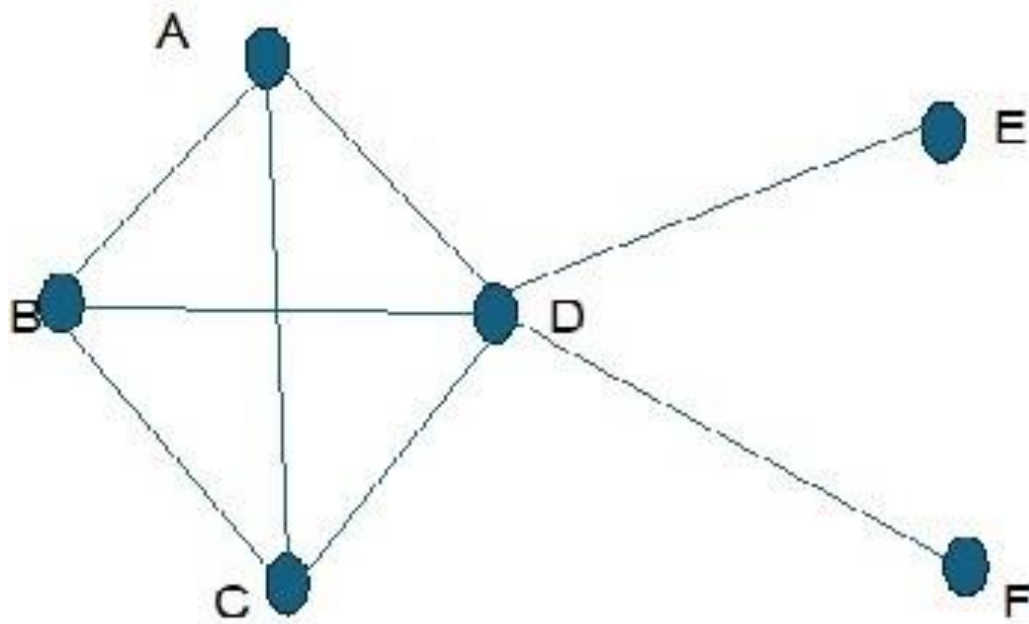
- (a)** Define Ring, Integral domain and Field. 03
- (b)** Define Transposition. For given permutations $A = (1\ 2\ 3\ 4\ 5)$ and $B = (2\ 3)(4\ 5)$, find AB . 04
- (c)** Construct the composition tables for (i) addition modulo 5 and (ii) multiplication modulo 5, for $\mathbb{Z}_5 = \{0, 1, 2, 3, 4\}$. Also check whether they have identity and inverse elements or not. 07
- Q.5 (a)** Draw a simple graph which has (i) 4 vertices with all having degree 3 and (ii) 3 vertices with all having degree 3. If graph for any one out of these 2 cases is not possible to draw then write the justification for that particular case. 03
- (b)** Define Regular graph and Complete graph. Is a regular graph always be a complete graph? If no then provide an example to justify this. 04
- (c)** Define Isomorphic graphs. Check whether the following graphs G_1 and G_2 are isomorphic or not by verifying all the necessary conditions. 07



OR

(a) Write the adjacency matrix for the given graph G.

03



(b) Define Euler graph and Hamiltonian graph and give one example for each. Also draw a graph (if possible) which is Euler but not Hamiltonian.

04

(c) Use Warshall's algorithm to find the shortest path between each pair of vertices for the given digraph.

07

