

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER-III (OLD) EXAMINATION – WINTER 2023

Subject Code:130001

Date:02-02-2024

Subject Name:Mathematics-III

Time:10:30 AM TO 01:30 PM

Total Marks:70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) Solve in series the equation $\frac{d^2y}{dx^2} + x^2y = 0$ 07

(b) (i) Solve $\frac{dy}{dx} + 2y \tan x = \sin x$ 03

(ii) Solve $(x^3 + 3xy^2)dx + (3x^2y + y^3)dy = 0$ 04

Q.2 (a) Using the method variation of parameter solve the differential equation $(D^2 + 4)y = \tan 2x$ 07

(b) Solve the equation $u_x = 2u_t + u$ given $u(x, 0) = 4e^{-4x}$, by the method of separation of variable. 07

OR

(b) Using the Frobenius method, obtain the series solution for $2x(x-1)y'' + (1-x)y' + 3y = 0$ about $x_0 = 0$ 07

Q.3 (a) Solve $(D^4 - 16)y = e^{2x} + x^4$ 07

(b) (i) Solve $p^2 + q^2 = npq$ 03

(ii) Solve $pz - qz = z^2 + (x+y)^2$ 04

OR

Q.3 (a) Solve $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x \cdot \sin(\log x)$ by Cauchy's linear equation 07

(b) (i) Form a partial differential equation by eliminating the arbitrary functions from $xyz = \phi(x+y+z)$ 03

(ii) Solve $(y+z)p + (z+x)q = x+y$ 04

Q.4 (a) Solve the initial value problem $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t$, 07

where $x(0) = 0$ & $x'(0) = 1$.

(b) (i) Find the inverse Laplace transform of $\log \frac{(s^2+b^2)}{(s^2+a^2)}$ 03

(ii) Find the Inverse Laplace transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ using Convolution theorem. 04

OR

Q.4 (a) Find the Inverse Laplace transform of $\frac{5s+3}{(s-1)(s^2+2s+5)}$ **07**

(b) (i) Find the Laplace transform of $e^{-3t}(2\cos 5t - 3\sin 5t)$. **03**

(ii) Find the Laplace transform of $\frac{e^{-at}-e^{-bt}}{t}$ **04**

Q.5 (a) (i) Define **03**

1. Gamma Function 2. Beta Function 3. Signum Function

(ii) Find half range *cosine* series for $f(x) = x$, $0 < x < 3$ **04**

(b) Obtain the Fourier series of $f(x) = \frac{1}{2}(\pi - x)$ in the interval **07**

$0 \leq x \leq 2\pi$ and hence deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

OR

Q.5 (a) (i) Find the fourier cosine integral of $f(x) = e^{-kx}$, where $x > 0, k > 0$ **03**

(ii) Find the Fourier series of $f(x) = x^2$ in the interval $(0, 4)$ **04**

(b) Find the Fourier series of $f(x) = \begin{cases} -\pi & -\pi < x < 0 \\ x & 0 < x < \pi \end{cases}$ **07**

Hence deduce that $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$
