

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV (OLD) EXAMINATION – WINTER 2023****Subject Code:140001****Date:11-01-2024****Subject Name:Mathematics-IV****Time:10:30 AM TO 01:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- | | Marks | | | | | | | | | | | | |
|---|--------------|--------|--------|--------|--------|------|---------|--------|--------|--------|--------|--------|--|
| Q.1 (a) Prove that
$(1 + \cos \theta + i \sin \theta)^n + (1 + \cos \theta - i \sin \theta)^n = 2^{n+1} \cos^n \frac{\theta}{2} \cos \frac{n\theta}{2}.$ | 07 | | | | | | | | | | | | |
| (b) Find a real root of the equation $3x = \cos x + 1$, correct to three decimal places by using Newton-Raphson method. | 07 | | | | | | | | | | | | |
| Q.2 (a) Find the all values of $(-1)^{\frac{1}{6}}$. | 07 | | | | | | | | | | | | |
| (b) Given that $u(x, y) = x^2 - y^2$ and $v(x, y) = -\left(\frac{y}{x^2 + y^2}\right)$. Prove that both u and v are harmonic functions, but $u + iv$ is not an analytic function. | 07 | | | | | | | | | | | | |
| OR | | | | | | | | | | | | | |
| (b) Show that the function $u(x, y) = x^4 - 6x^2y^2 + y^4$ is harmonic. Also find the analytic function $f(z) = u(x, y) + iv(x, y)$. | 07 | | | | | | | | | | | | |
| Q.3 (a) Define bilinear transformation. Find the bilinear transformation which maps $z_1 = 0, z_2 = 1, z_3 = \infty$ onto $w_1 = i, w_2 = -1, w_3 = -i$ respectively. | 07 | | | | | | | | | | | | |
| (b) Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along the path $y = x^2$. | 07 | | | | | | | | | | | | |
| OR | | | | | | | | | | | | | |
| Q.3 (a) Evaluate $\oint_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)(z-2)} dz$, where C is the circle $ z = 3$ by using Cauchy integral formula. | 07 | | | | | | | | | | | | |
| (b) Expand $\frac{1}{z^2 - 3z + 2}$ in the region
(1) $ z < 1$, (2) $1 < z < 2$, (3) $ z > 2$, (4) $0 < z - 1 < 1$. | 07 | | | | | | | | | | | | |
| Q.4 (a) Use false position method to find a root of the equation $x^2 - x - 2 = 0$ correct up to three decimal places. | 07 | | | | | | | | | | | | |
| (b) Find $f(0.12)$ by using Newton's interpolation for the following data: | 07 | | | | | | | | | | | | |
| <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>$x:$</td> <td>0.10</td> <td>0.15</td> <td>0.20</td> <td>0.25</td> <td>0.30</td> </tr> <tr> <td>$f(x):$</td> <td>0.1003</td> <td>0.1511</td> <td>0.2027</td> <td>0.2553</td> <td>0.3090</td> </tr> </table> | $x:$ | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | $f(x):$ | 0.1003 | 0.1511 | 0.2027 | 0.2553 | 0.3090 | |
| $x:$ | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | | | | | | | | |
| $f(x):$ | 0.1003 | 0.1511 | 0.2027 | 0.2553 | 0.3090 | | | | | | | | |
| OR | | | | | | | | | | | | | |
| Q.4 (a) Write the Trapezoidal rule. Find the value of $\int_0^6 \frac{e^x}{1+x} dx$ approximately by using Simpson's $\frac{3}{8}$ th rule with $h = 1$. | 07 | | | | | | | | | | | | |
| (b) Find a root of the equation $x^3 + x - 1 = 0$ correct to three decimal places, by using bisection method. | 07 | | | | | | | | | | | | |

Q.5 (a) Solve the following system of equation by Gauss-Seidel method correct upto two decimal places **07**

$$20x_1 + 2x_2 + x_3 = 30, x_1 - 40x_2 - 3x_3 = -75, 2x_1 - x_2 + 10x_3 = 30.$$

(b) Use forth order Runge-Kutta method to find the value of y when $x = 0.2$ given that $y' = x + y^2$ and $y = 1$ when $x = 0$. Take $h = 0.1$. **07**

OR

Q.5 (a) Apply Gauss elimination method to solve the following system of simultaneous linear equations: **07**

$$2x + y - z = 1, 5x + 2y + 2z = -4, 3x + y + z = 5$$

(b) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y(0) = 1$. Find y approximately at $x = 0.1$ by using Euler's method. Take $h = 0.02$. **07**
