

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-I (NEW) EXAMINATION – WINTER 2023****Subject Code:2110014****Date:11-01-2024****Subject Name:Calculus****Time:02:30 PM TO 05:30 PM****Total Marks:70****Instructions:**

1. Question No. 1 is compulsory. Attempt any four out of remaining Six questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) In each of the following choose the correct option. (MCQ)**Marks****07**

1. $\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} = \underline{\hspace{2cm}}$
(a) 2 (b) $\frac{1}{6}$ (c) -1 (d) 3
2. The sum of the series $2 + \frac{2}{3} + \frac{2}{9} + \frac{2}{27} + \dots$ is
(a) 3 (b) -3 (c) 4 (d) 5
3. The series $\sum_{n=1}^{\infty} \frac{1}{n^p} = \frac{1}{1^p} + \frac{1}{2^p} + \frac{1}{3^p} + \dots$ is divergent if
(a) $p > 1$ (b) $p = 1$ (c) $p \leq 1$ (d) none of these
4. If $f(x, y) = x^2 + 3xy + y - 1$ then the value of $\frac{\partial^2 f}{\partial x \partial y}$ at the point (1,2) is
(a) 4 (b) 3 (c) 2 (d) -6
5. If $u = e^{\frac{x}{y}}$ then the value of $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \underline{\hspace{2cm}}$
(a) -1 (b) 7 (c) 1 (d) 0
6. If u is a homogeneous function of x and y of degree n then the value of $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = \underline{\hspace{2cm}}$
(a) nu (b) n(n+1)u (c) n(n-1)u (d) 2nu
7. The value of $\int_0^1 \int_0^x e^{x+y} dy dx$ is
(a) 2e (b) e (c) $\frac{(e-1)^2}{2}$ (d) e - 1

(b) In each of the following choose the correct option. (MCQ)**07**

1. The value of the integral $\int_0^{\infty} \frac{1}{x^2+1} dx$ is _____
(a) π (b) $\frac{\pi}{2}$ (c) 2π (d) $\frac{\pi}{3}$
2. If $f(x, y) = 0$ represents an implicit function then $\frac{dy}{dx} = \underline{\hspace{2cm}}$
(a) $\frac{f_x}{f_y}$ (b) $\frac{f_{xx}}{f_{yy}}$ (c) $-\frac{f_x}{f_y}$ (d) 0
3. The volume of solid generated by revolving the graph of $y = x^2$ between $x = 1$ and $x = 2$ about the x-axis is _____
(a) $\frac{31}{5}\pi$ (b) $\frac{41}{5}\pi$ (c) $\frac{1}{5}\pi$ (d) $\frac{3}{5}\pi$

4. If $x = u + v$, $y = v - u$ then value of $\frac{\partial(x,y)}{\partial(u,v)}$ is _____
 (a) -1 (b) 2 (c) 3 (d) 4
5. $\lim_{n \rightarrow \infty} \frac{\sin n}{n} =$ _____
 (a) 0 (b) -2 (c) 1 (d) 2
6. The value of $\int_0^1 \int_0^1 \frac{dydx}{(1+x^2)(1+y^2)}$ is equal to
 (a) $\frac{\pi}{2}$ (b) $\frac{\pi^2}{16}$ (c) $\frac{\pi}{4}$ (d) π
7. The value of $\lim_{x \rightarrow 0} \cos x^{\cot x}$ is _____
 (a) 2 (b) -1 (c) 0 (d) 6
- Q.2** (a) Prove that $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots$ converges and find its sum. **03**
- (b) Test the convergence for $\sum_{n=1}^{\infty} ne^{-n^2}$ **04**
- (c) Find the value of $\sqrt{10}$ correct to four decimal places by using Taylor's series **07**
- Q.3** (a) Evaluate $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^2}$ **03**
- (b) Trace the curve $a^2x^2 = y^3(2a - y)$ **04**
- (c) Evaluate the following integrals
 (1) $\int_{-2}^2 \frac{dx}{x^2}$ **03**
 (2) $\int_0^1 \frac{dx}{(1-x)^{\frac{1}{3}}}$ **04**
- Q.4** (a) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y}{x^4+y^2}$ does not exist. **03**
- (b) Find the equations of the tangent plane and normal line to the surface $2xz^2 - 3xy - 4x = 7$ at $(1, -1, 2)$ **04**
- (c) Find the maximum and minimum values of $f(x, y) = x + y + \frac{1}{x} + \frac{1}{y}$ **07**
- Q.5** (a) If $z = e^{ax+by}f(ax - by)$ then prove that $b \frac{\partial z}{\partial x} + a \frac{\partial z}{\partial y} = 2abz$ **03**
- (b) If $z = f(x, y)$ and $x = e^u + e^{-v}$, $y = e^{-u} - e^v$ then prove that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y}$ **04**
- (c) (1) If $u = \sin^{-1} \left(\frac{x^2+y^2}{x+y} \right)$ then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \tan u$ **03**
 (2) If $u = \tan^{-1} \left(\frac{x^3+y^3}{x-y} \right)$ then prove that $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = 2\cos 3u \sin u$ **04**
- Q.6** (a) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{1-\sin \theta} r^2 \cos \theta dr d\theta$ **03**
- (b) Evaluate $\iint_R e^{ax+by} dydx$, R is the region bounded by $x = 0, y = 0, ax + by = 1$. **04**
- (c) Change the order of integration and Evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$ **07**

- Q.7**
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| (a) | Find the volume of a solid generated by revolving the region bounded by the curve $y = \sqrt{x}$, x-axis and the line $x=4$ about the y-axis, | 03 |
| (b) | Change the order of integration of $\int_0^\infty \int_x^\infty f(x,y) dx dy$ | 04 |
| (c) | Evaluate the integral $\int \int (y - x) dx dy$ over the region R in the xy-plane bounded by the straight lines $y=x-3$, $y=x+1$, $3y+x=5$, $3y+x=7$. | 07 |
