

Seat No.: _____

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GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV (NEW) EXAMINATION – WINTER 2023****Subject Code:3142008****Date:17-01-2024****Subject Name:Numerical Methods****Time:10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) Prove that $(1)(1 + \Delta)(1 - \nabla) = 1$ (2) $\nabla \equiv 1 - E^{-1}$. **03**

(b) Fit a straight line of the form $x = a + by$ to the data given below: **04**

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

(c) Find an Iterative formula for $\sqrt{N}$ where N is a positive integer and hence find $\sqrt{65}$. **07**

Q.2 (a) Using Newton's Forward Difference Method find the approximate value of $f(1.3)$ from the given table: **03**

x	1	2	3	4
$f(x)$	1.1	4.2	9.3	16.4

(b) Using Trapezoidal rule Evaluate $\int_0^2 \frac{x}{\sqrt{1+x^2}} dx$ with $n = 4$ **04**

(c) Solve the system of equations using Gauss Elimination method **07**

$$x + y + z = 7,$$

$$3x + 3y + 4z = 24,$$

$$2x + y + 3z = 16$$

OR

(c) Solve the following differential equation and find $y(0.1)$ and $y(0.2)$ **07**

$$\frac{dy}{dx} = 2x + y, \text{ with the fourth-order Runge-Kutta method, where } y(0) =$$

$$1, \text{ with } h = 0.1$$

- Q.3 (a)** Find Absolute error, Relative error, and Percentage error if $\frac{2}{3}$ is approximated to four Significant digits. **03**
- (b)** Find a root of the equation $x^3 - 4x - 9 = 0$ using the Bisection method in four stages. **04**
- (c)** Solve the following system of linear equations using Gauss-Seidel Method starting with (1,1,0) **07**
- $$25x + 2y - 3z = 48,$$
- $$3x + 27y - 2z = 56,$$
- $$x + 2y + 23z = 52$$

OR

- Q.3 (a)** Show that (1) $\Delta \equiv E \nabla \equiv \nabla E$ (2) $\Delta \nabla \equiv (\Delta - \nabla)$ **03**
- (b)** Find a Positive root of $x - e^{-x} = 0$ correct up to three decimal places using Regula Falsi Method. **04**
- (c)** Using Modified Euler's Method solve $\frac{dy}{dx} = x + y$ with initial condition $y(0) = 0$ choosing $h = 0.2$. Compute y for $x = 0.2, 0.4$. **07**

- Q.4 (a)** Write the formula for Runge-Kutta Fourth Order Method. **03**

- (b)** Find Inverse of the Matrix $\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$, Using Gauss Jordan Method. **04**

- (c)** Obtain the Cubic Spline in the interval $[0,2]$ for the following data with $M_0 = 0$ and $M_3 = -12$ **07**

x	0	2	4	6
y	1	9	41	41

OR

- Q.4 (a)** If $\pi = 3.141592$ is approximated as 3.14. Find the Absolute error, Relative error and Percentage error. **03**

- (b)** Fit the exponential curve $y = ax^b$ to the following data: **04**

x	20	16	10	11	14
y	22	41	120	89	56

- (c)** Find the Polynomial of degree 2 by Newton's forward difference Interpolation formula from the following table: **07**

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

- Q.5 (a)** Find the Third Divided difference with the arguments **03**
 $x = 2, 4, 9$ and 10 for the function $f(x) = x^3 - 2x$.
- (b)** Find the value of $y(4.4)$ using Milne's Predictor and Corrector Method **04**
 given $5xy' + y^2 - 2 = 0$ with $y(4) = 1, y(4.1) = 1.0049,$
 $y(4.2) = 1.0097, y(4.3) = 1.0143$.
- (c)** Find the largest Eigen value of $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$ consider $x_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ **07**
 By Power Method and find the corresponding Eigen vector.

OR

- Q.5 (a)** Evaluate $\int_0^6 \frac{1}{1+x} dx$ taking $h = 1$ using Simpson's 1/3 rule. Hence obtain **03**
 an approximate value of $\log 7$.
- (b)** Using Secant Method find the solution of $f(x) = xe^x - 1 = 0$ correct up **04**
 to three decimal places between 0 and 1.
- (c)** By Using Lagrange's formula find $y(2)$ for the following: **07**

x	-1	0	1	3
y	2	1	0	-1
