

GUJARAT TECHNOLOGICAL UNIVERSITY**BE - SEMESTER-IV (NEW) EXAMINATION – WINTER 2023****Subject Code:3144202****Date:01-02-2024****Subject Name: Discrete Mathematics and Graph Theory****Time: 10:30 AM TO 01:00 PM****Total Marks:70****Instructions:**

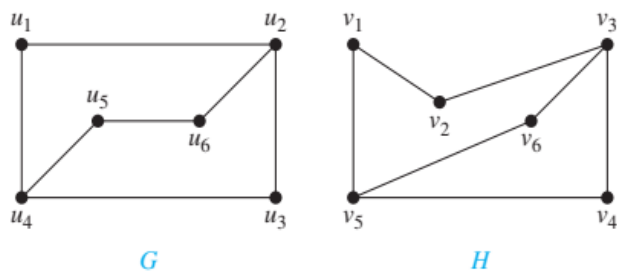
1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- Q.1 (a)** Construct a truth table for each of these compound propositions **03**
 $(p \vee \neg p) \rightarrow q$
- (b)** Let $f, g, h: R \rightarrow R$ be defined as: **04**
 $f(x) = 2x^3 + 5, \quad g(x) = \cos x$ and $h(x) = x^3 - 1$
 Find $ho(gof)$ and $(hog)of$. Are they equal?
- (c)** Let $U = \{a, b, c, d, e, f, g, h, p, q, r\}$ **07**
 $X = \{a, b, c, d, e\}$
 $Y = \{c, d, e, f, g, h\}$
 $Z = \{h, p, q, r\}$
 Compute $X \cup Y, Y \cap Z \cap X, X - (Y \cup Z), (X \cup Y)', X', X' \cap Y',$
 $X \Delta Y$ Using Venn Diagram. (Note: $X \Delta Y = (X - Y) \cup (Y - X)$)
- Q.2 (a)** Verify that the relation $x \equiv y(mod m)$ defined on the set of integers is an **03**
 equivalence relation..
- (b)** Obtain the d.n.f. of the form $(p \rightarrow q) \wedge (\neg p \wedge q)$ **04**
- (c)** Determine the validity of the argument given: **07**
 If I study, then I will not fail in Discrete Mathematics.
 If I do not play cricket, then I will study.
 But I failed in Discrete Mathematics.

 Therefore I must have played cricket.
OR
- (c)** Determine the validity of argument given: **07**
 S_1 : If a man is a bachelor, he is unhappy.
 S_2 : If a man is unhappy, he dies young.

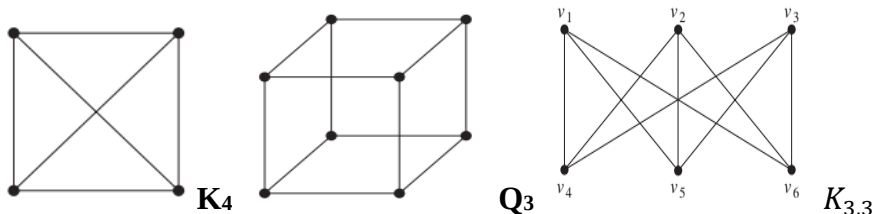
 S : Bachelors die young.
- Q.3 (a)** Define lattice. Determine whether POSET $\{\{1, 2, 3, 4, 5\}; |\}$ is lattice **03**
- (b)** Let $A = \{a, b, c\}$ and $P(A)$ its power set. Let $\subseteq$ be the inclusion relation on **04**
 the elements of $P(A)$. Draw Hass diagram for $\langle P(A), \subseteq \rangle$
- (c)** Find the transitive closure of $R = \{(1,2), (3,4), (4,5), (4,1), (1,1)\}$. **07**
 Where, $A = \{1,2,3,4,5\}$.
- OR**
- Q.3 (a)** Solve the recurrence relation: $a_{n+2} - 5a_{n+1} + 6a_n = 2$. **03**
- (b)** A graph G has 15 edges, 3 vertices of degree 4 and other vertices of **04**
 degree 3. Find the number of vertices in G.
- (c)** Let A be a set of factors of positive integer m and relation is divisibility on **07**
 A. For $m = 45$, show that POSET $(A, \leq)$ is a Lattice

- Q.4** (a) Define the following by drawing graphs (i) weak component (ii) unilateral component (iii) strong component. **03**
 (b) Define complement of a sub graph by drawing the graphs. **04**
 (c) Determine whether the graphs G and H displayed in figure are isomorphic. **07**

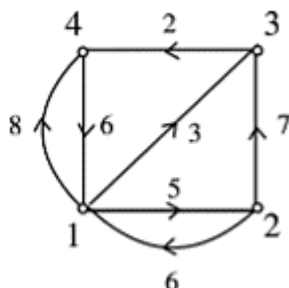


OR

- Q.4** (a) Explain the following terms with proper illustrations. (a) Directed graphs **03**
 (b) Simple and elementary path (c) Reachability of a vertex **04**
 (b) Define Planar Graph. Are K_4 , Q_3 , $K_{3,3}$ planar? Explain. **04**



- (c) Find the shortest path between each pair of vertices for a simple using Warshall's Algorithm digraph. **07**



- Q.5** (a) Show that the set of cube roots of unity forms a group under multiplication. **03**
 (b) Define group permutation. Find the inverse of the permutation **04**

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$$

- (c) Show that $(\mathbb{Z}_4, +_4)$ is a cyclic group and find all the subgroups of $(\mathbb{Z}_4, +_4)$. **07**

OR

- Q.5** (a) Define subgroup and group Homomorphism **03**
 (b) Let $(H_1, *)$ and $(H_2, *)$ be subgroups of a group $(G, *)$. Show that $(H_1 \cap H_2, *)$ is also a subgroup of $(G, *)$. Will $(H_1 \cup H_2, *)$ be also a subgroup of $(G, *)$? **04**

- (c) Show that an algebraic structure $(G, \bullet)$ is an abelian group, **07**

Where $G = \{A, B, C, D\}$, $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$,
 $D = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ and $\bullet$ is the binary operation of matrix multiplication.
