

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-IV (OLD) EXAMINATION – WINTER 2024****Subject Code:140001****Date:19-11-2024****Subject Name: Mathematics-IV****Time:02:30 PM TO 05:30 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Q.1 (a) Find the fifth root of unity. **07**

(b) Check whether $f(z)$ is continuous at $z = 0$ if $f(z) = \begin{cases} \frac{\operatorname{Re}(z^2)}{|z|}, & z \neq 0 \\ 0, & z = 0 \end{cases}$ **07**

Q.2 (a) Determine the analytic function whose real part is $e^{-x}(x \cos y - y \sin y)$, using Milne-Thompson's Method. **07**

(b) Evaluate $\int_C (x^2 - iy^2) dz$ along the parabola $y = 2x^2$ from $(1,2)$ to $(2,8)$. **07**

OR

(b) Evaluate $\int_C \frac{5z+7}{z^2+2z-3} dz$ where C is the circle $|z-2| = 2$. **07**

Q.3 (a) Expand $f(z) = \sin z$ in Taylor series about $z = 0$. **07**

(b) Find the Laurent's series expansion of $f(z) = \frac{7z-2}{(z+1)z(z-2)}$ in the region $1 < |z| < 2$. **07**

OR

Q.3 (a) Using Residue theorem, evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$, where C is the circle $|z| = 3$. **07**

(b) Find the bilinear transformation which maps the points $1, -1, \infty$ onto the points $1+i, 1-i, 1$ respectively. **07**

Q.4 (a) Write $f(x) = x^4 - 2x^3 + x^2 - 2x + 1$ in factorial notation and find $\Delta^4 f(x)$. **07**

(b) Using Newton's forward formula, find the value of $f(1.6)$ if **07**

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.50

OR

Q.4 (a) Using Simpson's 1/3 rule, evaluate $\int_0^6 f(x) dx$ from the following data. Take $h = 1$. **07**

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.3333	0.25	0.2	0.1666	0.1428

(b) State Trapezoidal rule with $n = 10$ and evaluate $\int_0^1 e^x dx$. **07**

Q.5 (a) Solve the following system of equations using Gauss Elimination method. **07**

$$8y + 2z = -7$$

$$3x + 5y + 2z = 8$$

$$6x + 2y + 8z = 26$$

(b) Compute the real root of $f(x) = x - 2\sin x = 0$, correct upto 6 decimal places **07**
using Secant method, starting from $x_0 = 2, x_1 = 1.9$.

OR

Q.5 (a) Use power method to find the largest eigen value of the matrix $A = \begin{bmatrix} 3 & -5 \\ -2 & 4 \end{bmatrix}$. **07**

(b) Using Euler's method to find $y(1.4)$ given that $\frac{dy}{dx} = xy^{1/2}$, $y(1) = 1$. **07**
