

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-IV (NEW) EXAMINATION – WINTER 2024****Subject Code:3142008****Date:21-11-2024****Subject Name: Numerical Methods****Time:02:30 PM TO 05:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

Marks

Q.1 (a) Using Simpson's 3/8 rule, evaluate $\int_0^1 \frac{dx}{1+x^2}$ by taking $h = \frac{1}{6}$. **03**

(b) Using Newton's forward interpolation, find the value of $f(1.6)$. **04**

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5

(c) Solve the following equations by Gauss-Jacobi method: **07**
 $4x + y + 3z = 17$, $x + 5y + z = 14$, $2x - y + 8z = 12$.

Q.2 (a) Prove that $\Delta \equiv E \nabla \equiv \nabla E$. **03**

(b) Find a real root of the equation $3x = \cos x + 1$ using Newton Raphson method correct up to 4 decimal places. **04**

(c) Find the negative root of equation $x^3 - 7x + 3 = 0$ using Bisection method, correct up to 3 decimal places. **07**

OR

(c) Find root of an equation $xe^x - 1 = 0$ using Secant method correct to 4 significant digits. **07**

Q.3 (a) Using method of least squares, find the best fitting straight line to the following data: **03**

x	1	2	3	4	5
y	1	3	5	6	5

(b) Determine the value of y using Euler's method when $x = 0.2$ given that $y(0) = 1$ and $\frac{dy}{dx} = y - \frac{2x}{y}$. Choose the step size $h = 0.1$. **04**

(c) Apply fourth order Runge-Kutta method to calculate $y(0.2)$ with $h = 0.1$ given that **07**

$$y' = x + y, y(0) = 1$$

OR

Q.3 (a) Find the dominant Eigen value of $A = \begin{bmatrix} 2 & 3 \\ 5 & 4 \end{bmatrix}$ by using power method and the corresponding Eigen vector. **03**

(b) Use improve Euler's methods to obtain an approximate value of $y(0.1)$ for the equation **04**

$$\frac{dy}{dx} = x^2 + y, y(0) = 1, \text{ taking } h = 0.05.$$

(c) Solve the boundary value problem $xy'' + y = 0$ with $y(1) = 1, y(2) = 2$, take $n = 4$ by finite difference method. **07**

Q.4 (a) Compute $\int_0^1 \frac{dx}{x+1}$ using Gaussian formula for $n = 3$. **03**

(b) Given that **04**

x	1	1.2	1.4	1.6	1.8	2	2.2
y	2.7183	3.3201	4.0552	4.9530	6.0496	7.3891	9.0250

Find $\frac{d^2y}{dx^2}$ at $x = 2.2$.

(c) Solve the following systems by Gauss elimination method. **07**

$$2x + y - z = 1, \quad 5x + 2y + 2z = -4, \quad 3x + y + z = 5.$$

OR

Q.4 (a) Evaluate $\int_4^{5.2} \log x \, dx$ by using Trapezoidal rule taking 6 subintervals. **03**

(b) Following data gives corresponding values of pressure and volume of a super-heated steam: **04**

V	2	4	6	8	10
P	105	42.7	25.3	16.7	13

Find the rate of change of pressure with respect to volume when $V = 2$.

(c) Solve the following equations by Gauss-Seidel method: **07**

$$5x + y - z = 10, \quad 2x + 4y + z = 14, \quad x + y + 8z = 20.$$

Q.5 (a) Fit the curve $y = ae^{bx}$ to the following data: **03**

x	0	2	4
y	5.012	10	31.62

(b) Find the percentage error in calculating the area of a rectangle when an error of 3% is made in measuring each of its sides. **04**

(c) Using Newton divided difference formula, find $f(8)$ from the following table: **07**

x	4	5	7	10	11	13
$y = f(x)$	48	100	244	900	1210	2028

OR

Q.5 (a) Compute $f(2)$ by using Lagrange's interpolation formula. **03**

x	-1	0	1	3
$y = f(x)$	2	1	0	-1

(b) Discuss type of errors. **04**

(c) Fit a second degree parabola to the following data: **07**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

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