

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-IV (NEW) EXAMINATION – WINTER 2024****Subject Code:3144202****Date:03-12-2024****Subject Name:Discrete Mathematics and Graph Theory****Time:02:30 PM TO 05:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

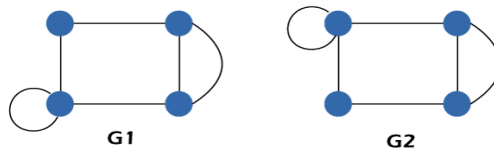
		Marks
Q.1	(a) Prove the following using Venn diagram. $A \cap (B \oplus C) = (A \cap B) \oplus (A \cap C)$	03
	(b) How many four digit numbers can be formed out of the digits 1, 2, 3, 4, 5, 7, 8, 9 If (1) No repetition is permitted. (2) How many of these will greater than 3000?	04
	(c) Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study physics and biology, 30 do not study any of three subjects: (1) Find the number of students studying all three subjects. (2) Find the number of students studying exactly one of the three subjects.	07
Q.2	(a) Define one-one mapping using suitable example.	03
	(b) Three functions F, G, H are defined from $R \rightarrow R$ as follows $F(x) = 2x^3 + 5$, $g(x) = \cos x$, $h(x) = x^3 - 1$ find (1) $ho(gof)$ (2) $(hog)of$	04
	(c) Obtain disjunction normal form of (1) $(p \rightarrow q) \wedge (\sim p \wedge q)$ (2) $(p \wedge (p \rightarrow q)) \rightarrow q$	07
	OR	
	(c) Obtain the conjunction normal form of each of the following. (1) $p \wedge (p \rightarrow q)$ (2) $\sim (p \vee q) \leftrightarrow (p \wedge q)$	07
Q.3	(a) If $A = \{1, 2, 3, 4, 5\}$ and $R = \{(1, 2), (3, 4), (4, 5), (4, 1), (1, 1)\}$ find its transitive closure.	03
	(b) Let R be the relation on the set A. $A = \{5, 6, 8, 10, 28, 36, 48\}$. Let $R = \{(a,b) / a \text{ is divisor of } b\}$. Draw the hasse diagram. Compare with diagraph. Determine whether R is equivalence relation.	04
	(c) Let A be the set of factors of positive integer m and relation is divisibility on A. i.e. $R = \{(x,y) / x,y \in A, x \text{ divides } y\}$ For $m = 45$ show that poset $(A, \leq)$ is Lattice. Draw hasse diagram and give join and meet for the lattice.	07
	OR	
Q.3	(a) Find its transitive closure R by Warshall's algorithm. Where $A = \{1, 2, 3, 4, 5\}$. $R = \{(1,1),(1,2),(1,3),(1,4), (3,1),(3,2), (5,1), (5,2), (5,3), (5,4), (5,5)\}$	03
	(b) Let $S = \{1, 2, 3\}$ $(P(s), \subseteq)$ is a POSET. And draw diagram of POSET.	04

- (c) Let n be the positive integer S_n , be the set of all divisors of n , Let D denotes the relation of 'division'. Draw the diagram of lattice for (1) $n = 24$, (2) $n = 30$, (3) $n = 6$. 07
- Q.4** (a) The set of Q_1 of all rational number other than 1 with operation $*$ defined by $a*b = a+b-ab$ is an abelian group. 03
- (b) Let $G = \{1, 2, 3, 4, 5, 6\}$, X_8 is group? Justify your answer. 04
- (c) Is set M of 2×2 matrices where entries of matrices are real number form a ring with unity with respect to addition and multiplication? 07

OR

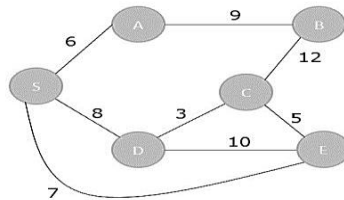
- Q.4** (a) Let G be the set of all nonzero real number and $*$ a binary operation on G , defined by, $a*b = \frac{ab}{2}$ is an abelian group. 03
- (b) Justify $\langle Z_6, + \rangle$ is abelian group? 04
- (c) Consider the set of $(C, +, \bullet)$, Where C is the set of complex numbers and $+$ and $\bullet$ are ordinary addition and multiplication. Show that $(C, +, \bullet)$ ring. 07

- Q.5** (a) Define regular graph and Complete graph. Is every regular graph is Complete? 03
- (b) 04



Is the above are isomorphic?

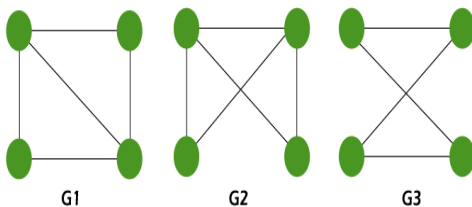
- (c) 07



Using Dijkstra's algorithm, find the shortest path between vertices S to E .

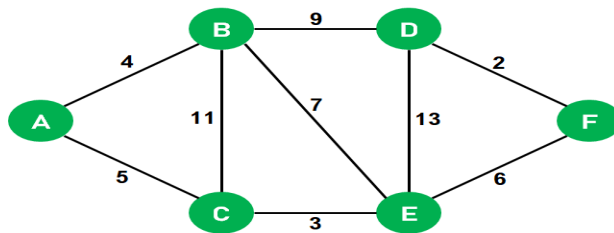
OR

- Q.5** (a) Define Bipartite graph and Complete bipartite graph with one example. 03
- (b) 04



Is the above are isomorphic?

- (c) 07



Using Dijkstra's algorithm, find the shortest path between vertices A to F .
