

Seat No. / Enrolment No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Diploma Engineering – SEMESTER – 2 (OLD) – EXAMINATION – Winter-2024

Subject Code: 3320002

Date: 04-01-2025

Subject Name: ADVANCED MATHEMATICS (GROUP-1)

Time: 10:30 AM TO 01:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of simple calculators and non-programmable scientific calculators are permitted.
5. English version is authentic.

Q.1- Fill in the blanks using appropriate choice from the given options: (14)

(યોગ્ય વિકલ્પ પસંદ કરી ખાલી જગ્યા પૂરો.)

(1) The inverse of complex number $3+4i =$ _____.

- (a) $\frac{3}{25} - \frac{4}{25}i$ (b) $\frac{3}{25} + \frac{4}{25}i$ (c) $-\frac{3}{25} - \frac{4}{25}i$ (d) $-\frac{3}{25} + \frac{4}{25}i$

(૧) સંકર સંખ્યા $3+4i$ નો વ્યસ્ત _____ છે.

- (a) $\frac{3}{25} - \frac{4}{25}i$ (b) $\frac{3}{25} + \frac{4}{25}i$ (c) $-\frac{3}{25} - \frac{4}{25}i$ (d) $-\frac{3}{25} + \frac{4}{25}i$

(2) $\sqrt{-9} =$ _____.

- (a) $3i$ (b) $-3i$ (c) $\pm 3i$ (d) $3i^2$

(૨) $\sqrt{-9} =$ _____.

- (a) $3i$ (b) $-3i$ (c) $\pm 3i$ (d) $3i^2$

(3) $i^{998} =$ _____.

- (a) i (b) -1 (c) 1 (d) $-i$

(૩) $i^{998} =$ _____.

- (a) i (b) -1 (c) 1 (d) $-i$

(4) If $Z = 5i - 3$ then $\bar{Z} =$ _____.

- (a) $5i + 3$ (b) $5i - 3$ (c) $-5i + 3$ (d) $-5i - 3$

(૪) જો $Z = 5i - 3$ હોય તો $\bar{Z} =$ _____.

- (a) $5i + 3$ (b) $5i - 3$ (c) $-5i + 3$ (d) $-5i - 3$

(5) If $f(x) = \sin x$ then $f\left(\frac{\pi}{2} + x\right) =$ _____.

- (a) $\cos x$ (b) $-\cos x$ (c) $\sin x$ (d) $-\sin x$

(૫) જો $f(x) = \sin x$ હોય તો $f\left(\frac{\pi}{2} + x\right) =$ _____.

- (a) $\cos x$ (b) $-\cos x$ (c) $\sin x$ (d) $-\sin x$

(6) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 1}{2x + 2} =$ _____.

- (a) 0 (b) ∞ (c) $\frac{1}{2}$ (d) 2

(૬) $\lim_{x \rightarrow 1} \frac{x^2 + 2x - 1}{2x + 2} =$ _____.

- (a) 0 (b) ∞ (c) $\frac{1}{2}$ (d) 2

(7) If $y = x^2$ then $\frac{d^2y}{dx^2} =$ _____.

- (a) 2 (b) $2x$ (c) 0 (d) x^2

(૭) જો $y = x^2$ હોય તો $\frac{d^2y}{dx^2} =$ _____.

- (a) 2 (b) $2x$ (c) 0 (d) x^2

(8) $\frac{d}{dx} \tan^{-1}\left(\frac{x}{a}\right) =$ _____.

- (a) $\frac{x}{x^2 + a^2}$ (b) $\frac{a}{x^2 + a^2}$ (c) $\frac{1}{x^2 + a^2}$ (d) $\frac{a^2}{x^2 + a^2}$

(૮) $\frac{d}{dx} \tan^{-1}\left(\frac{x}{a}\right) =$ _____.

(a) $\frac{x}{x^2+a^2}$ (b) $\frac{a}{x^2+a^2}$ (c) $\frac{1}{x^2+a^2}$ (d) $\frac{a^2}{x^2+a^2}$

(9) $\frac{d}{dx}(\log \cos x) = \underline{\hspace{2cm}}$.

(a) $\sec x$ (b) $\tan x$ (c) $-\tan x$ (d) $\cot x$

(10) $\frac{d}{dx}(\log \cos x) = \underline{\hspace{2cm}}$.

(a) $\sec x$ (b) $\tan x$ (c) $-\tan x$ (d) $\cot x$

(10) The equation of motion of a particle is $s = t^3 - 2t^2 + 3t + 4$ the velocity at $t = 2$ is _____ units.

(a) 7 (b) 4 (c) 6 (d) -6

(10) કણની ગતિનું સમીકરણ $s = t^3 - 2t^2 + 3t + 4$ છે તો $t = 2$ સેકન્ડે તેનો વેગ _____ એકમ થાય.

(a) 7 (b) 4 (c) 6 (d) -6

(11) $\int \cos(5x+3)dx = \underline{\hspace{2cm}} + C$.

(a) $\frac{1}{5}\cos(5x+3)$ (b) $\frac{1}{5}\sin(5x+3)$ (c) $\sin(5x+3)$ (d) $-\sin(5x+3)$

(11) $\int \cos(5x+3)dx = \underline{\hspace{2cm}} + C$.

(a) $\frac{1}{5}\cos(5x+3)$ (b) $\frac{1}{5}\sin(5x+3)$ (c) $\sin(5x+3)$ (d) $-\sin(5x+3)$

(12) $\int \frac{f'(x)}{f(x)}dx = \underline{\hspace{2cm}}$.

(a) $\log[f(x) + f'(x)] + c$ (b) $\log[f(x)] + c$ (c) $\log[f'(x)] + c$ (d) $n[f(x)]^{n-1} + c$

(12) $\int \frac{f'(x)}{f(x)}dx = \underline{\hspace{2cm}}$.

(a) $\log[f(x) + f'(x)] + c$ (b) $\log[f(x)] + c$ (c) $\log[f'(x)] + c$ (d) $n[f(x)]^{n-1} + c$

(13) The order of a differential equation $\left(\frac{d^4y}{dx^4}\right)^2 - \left(\frac{d^2y}{dx^2}\right)^3 + 2y = 0$ is _____.

- (a) 2 (b) 4 (c) 3 (d) 1

(૧૩) વિકલ સમીકરણ $\left(\frac{d^4y}{dx^4}\right)^2 - \left(\frac{d^2y}{dx^2}\right)^3 + 2y = 0$ માટે કક્ષા _____ છે.

- (a) 2 (b) 4 (c) 3 (d) 1

(14) An Integrating factor of the differential equation $\frac{dy}{dx} + y = 2x$ is _____.

- (a) e^x (b) 1 (c) 2 (d) $\log x$

(૧૪) વિકલ સમીકરણ $\frac{dy}{dx} + y = 2x$ માટે, સંકલ્પકારક અવયવ _____ છે.

- (a) e^x (b) 1 (c) 2 (d) $\log x$

Q-2 (A) Attempt any two (કોઇપણ બેના જવાબ આપો) (6)

(1) Find inverse complex numbers of $\frac{4+3i}{5-3i}$.

(૧) સંકર સંખ્યા $\frac{4+3i}{5-3i}$ નો વ્યસ્ત શોધો.

(2) Find square root of $Z = 3 - 4\sqrt{10}i$.

(૨) $Z = 3 - 4\sqrt{10}i$ નું વર્ગમૂળ શોધો.

(3) For $Z = 1 - i$, find $|\bar{Z}|$ and $\arg(Z)$.

(૩) $Z = 1 - i$, માટે $|\bar{Z}|$ અને $\arg(Z)$ શોધો.

(B) Attempt any two (કોઇપણ બેના જવાબ આપો) (8)

(1) Find $\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 + x + 2}{x^2 + x - 2}$

(૧) $\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 + x + 2}{x^2 + x - 2}$ શોધો.

(2) Evaluate $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

(૨) કિમત શોધો: $\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}$

(૩) If $f(x) = \log\left(\frac{1-x}{1+x}\right)$ then prove that $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.

(૩) જો $f(x) = \log\left(\frac{1-x}{1+x}\right)$ હોય તો સાબિત કરો કે $f\left(\frac{2x}{1+x^2}\right) = 2f(x)$.

Q-3 (A) Attempt any two (કોઇપણ બેના જવાબ આપો) (6)

(1) Differentiate $f(x) = x^2$ with respect to x by using the definition of derivative.

(૧) વિકલનની વ્યાખ્યાની મદદથી $f(x) = x^2$ નું x ની સાપેક્ષે વિકલન કરો.

(2) Find $\frac{dy}{dx}$ of $\frac{3+2\sin x}{3\sin x+2}$.

(૨) $\frac{3+2\sin x}{3\sin x+2}$ માટે $\frac{dy}{dx}$ મેળવો.

(3) If $y = \sin x^x$ then find $\frac{dy}{dx}$.

(૩) જો $y = \sin x^x$ હોય તો $\frac{dy}{dx}$ મેળવો.

(B) Attempt any two (કોઇપણ બેના જવાબ આપો) (8)

(1) If $y = A \cos pt + B \sin pt$ then prove that $\frac{d^2y}{dx^2} + p^2y = 0$.

(૧) જો $y = A \cos pt + B \sin pt$ હોય તો સાબિત કરો કે $\frac{d^2y}{dx^2} + p^2y = 0$ થાય.

(2) Find Maximum and Minimum value of $f(x) = 2x^3 - 15x^2 + 36x + 10$.

(૨) વિધેય $f(x) = 2x^3 - 15x^2 + 36x + 10$ માટે મહત્તમ અને ન્યૂનતમ કિમતો મેળવો.

(3) The equation of motion of a particle is $s = t^3 + 2t^2 - 3t + 5$. Find the velocity and acceleration of the particle at $t = 1$ and $t = 2$ seconds.

(૩) કણની ગતિનું સમીકરણ $s = t^3 + 2t^2 - 3t + 5$ છે તો $t = 1$ અને $t = 2$ સેકન્ડે કણનો વેગ અને પ્રવેગ મેળવો.

Q-4 (A) Attempt any two (કોઇપણ બેના જવાબ આપો)

(6)

(1) Evaluate $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$

(૧) કિંમત શોધો: $\int \frac{\cos 2x}{\sin^2 x \cdot \cos^2 x} dx$

(2) Evaluate $\int x \cdot e^x dx$

(૨) કિંમત શોધો: $\int x \cdot e^x dx$

(3) Evaluate $\int \frac{\sin(\log x)}{x} dx$

(૩) કિંમત શોધો: $\int \frac{\sin(\log x)}{x} dx$

(B) Attempt any two (કોઇપણ બેના જવાબ આપો)

(8)

(1) Find $\int \frac{x^2}{1+x^6} dx$

(૧) $\int \frac{x^2}{1+x^6} dx$ મેળવો.

(2) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$

(૨) કિંમત શોધો: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\cot x}}{\sqrt{\tan x} + \sqrt{\cot x}} dx$

(3) Find the area enclosed by the parabola $y = x^2$ and the line $y = x + 2$.

(૩) વક્ર $y = x^2$ અને રેખા $y = x + 2$ થી સીમિત પ્રદેશનું ક્ષેત્રફળ શોધો.

Q-5 (A) Attempt any two (કોઇપણ બેના જવાબ આપો)

(6)

(1) Find $\lim_{n \rightarrow \infty} 4 \left[\frac{1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3}{n^4} \right]$.

(૧) $\lim_{n \rightarrow \infty} 4 \left[\frac{1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3}{n^4} \right]$ મેળવો.

(2) Solve the differential equation $xdy + ydx = 0$.

(૨) વિકલ સમીકરણ ઉકેલો : $xdy + ydx = 0$.

(3) Solve the differential equation $4\frac{dy}{dx} + 8y = 5e^{-3x}$.

(૩) વિકલ સમીકરણ ઉકેલો : $4\frac{dy}{dx} + 8y = 5e^{-3x}$.

(B) Attempt any two (કોઈપણ બેના જવાબ આપો)

(8)

(1) Solve: $x\frac{dy}{dx} = x + y$.

(૧) ઉકેલો : $x\frac{dy}{dx} = x + y$.

(2) Solve: $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$.

(૨) ઉકેલો : $\frac{dy}{dx} = \frac{y}{x} + \sin\left(\frac{y}{x}\right)$.

(3) Solve : $\frac{dy}{dx} - \frac{2xy}{1+x^2} = x^2 + 1$

(૩) ઉકેલો : $\frac{dy}{dx} - \frac{2xy}{1+x^2} = x^2 + 1$