

Enrolment No./Seat No _____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER-IV EXAMINATION – WINTER 2025

Subject Code: 140001

Date: 28-11-2025

Subject Name: Mathematics-IV

Time: 02:30 PM TO 05:30 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

- | | Marks | | | | | | | | | | | | | | |
|--|-----------|-----|----|-----|-----|-----|---|---|--------|---|----|----|-----|-----|-----|
| Q.1 (a) Find the fifth root of unity. | 07 | | | | | | | | | | | | | | |
| (b) Determine polynomial using Newton divided difference interpolation formula for the data. | 07 | | | | | | | | | | | | | | |
| <table border="1" style="margin-left: auto; margin-right: auto;"><tr><td>x</td><td>0</td><td>2</td><td>3</td><td>4</td><td>7</td><td>9</td></tr><tr><td>$f(x)$</td><td>4</td><td>26</td><td>58</td><td>112</td><td>466</td><td>922</td></tr></table> | | x | 0 | 2 | 3 | 4 | 7 | 9 | $f(x)$ | 4 | 26 | 58 | 112 | 466 | 922 |
| x | 0 | 2 | 3 | 4 | 7 | 9 | | | | | | | | | |
| $f(x)$ | 4 | 26 | 58 | 112 | 466 | 922 | | | | | | | | | |
| Q.2 (a) Evaluate by Cauchy's integral formula, | 07 | | | | | | | | | | | | | | |
| $\int_C \frac{e^{z^2}}{(z+i)^4} dz, \text{ where } C: z+i = 2.$ | | | | | | | | | | | | | | | |
| (b) Find the value of $\int_0^{2+i} (\bar{z})^2 dz$ along the real axis from 0 to 2 and then vertically from 2 to $2+i$. | 07 | | | | | | | | | | | | | | |
| OR | | | | | | | | | | | | | | | |
| (b) Show that the function $v(x, y) = x^3y - xy^3 + xy + x + y$ is harmonic. Determine the analytic function $f(z)$. | 07 | | | | | | | | | | | | | | |
| Q.3 (a) Define Mobius transformation. Find the Mobius transformation which maps the points $i, -1, 1$ of the z -plane into the points $0, 1, \infty$ in w -plane respectively. | 07 | | | | | | | | | | | | | | |
| (b) Find the Laurent series in powers of z that represent $f(z) = \frac{1}{2} \left[\frac{1}{(z+2)} - \frac{1}{(z+4)} \right]$ for (i) $ z < 2$ (ii) $2 < z < 4$. | 07 | | | | | | | | | | | | | | |
| OR | | | | | | | | | | | | | | | |
| Q.3 (a) Evaluate | 07 | | | | | | | | | | | | | | |
| $\int_0^{2\pi} \frac{\cos 3\theta}{5 - 4 \cos \theta} d\theta.$ | | | | | | | | | | | | | | | |
| (b) Using Residue Theorem, evaluate | 07 | | | | | | | | | | | | | | |
| $\int_C \frac{z^2}{(z-1)^2(z+2)} dz,$ | | | | | | | | | | | | | | | |
| where C is the circle $ z = 3$. | | | | | | | | | | | | | | | |

Q.4 (a) Find a real root of $x^3 - 9x + 1 = 0$ correct to four decimal places using Regula-Falsi method. **07**

(b) Determine polynomial of degree 3 using Lagrange's interpolation for the data. **07**
Also find $f(2)$.

x	-1	0	1	3
$f(x)$	2	1	0	-1

OR

Q.4 (a) Use power method to find the dominant eigen value and eigen vector of the matrix **07**

$$\begin{bmatrix} 1 & -3 & 2 \\ 4 & 4 & -1 \\ 6 & 3 & 5 \end{bmatrix}$$

(b) Write Simpson's $\frac{1}{3}$ rule. Evaluate $\int_0^{\pi} \frac{\sin^2 x}{5+4 \cos x} dx$ with $n = 6$ by using Simpson's $\frac{3}{8}$ rule. **07**

Q.5 (a) Use Gauss-Jacobi method to solve **07**

$$20x + y - 2z = 17,$$

$$3x + 20y - z = -18$$

$$2x - 3y + 20z = 25.$$

(b) Using Euler's method, find y at $x = 1$ for differential equation **07**
 $y' = x^2 + y^2, y(0) = 1$ by taking $h = 0.2$.

OR

Q.5 (a) Solve the following system by using Gauss elimination method with partial pivoting **07**

$$x + y + z = 7,$$

$$3x + 3y + 4z = 24$$

$$2x + y + 3z = 16.$$

(b) Using the fourth order Range-Kutta method, find y at $x = 0.1$ for **07**
differential equation $y' = 3e^x + 2y, y(0) = 0$ by taking $h = 0.1$.
