

GUJARAT TECHNOLOGICAL UNIVERSITY**BE- SEMESTER-IV EXAMINATION – WINTER 2025****Subject Code:3142008****Date:15-11-2025****Subject Name:Numerical Methods****Time:02:30 PM TO 05:00 PM****Total Marks:70****Instructions:**

1. Attempt all questions.
2. Make suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Simple and non-programmable scientific calculators are allowed.

MARKS

- Q.1*** (a) Find a real positive root of the equation $x \sin(x) + \cos(x) = 0$ which is near $x = \pi$ correct to four significant digits using Newton Raphson method. **03**
- (b) Solve the given system of equations using Gauss elimination method. **04**
 $x + y + z = 7$, $3x + 3y + 4z = 24$, $2x + y + 3z = 16$
- (c) Fit a second-degree parabola $y = a + bx + cx^2$ to the following data: **07**

x	0	1	2	3	4
y	1	1.8	1.3	2.5	6.3

- Q.2** (a) Evaluate the integral $\int_0^1 \frac{1}{1+x^2} dx$ using Simpson's $\frac{1}{3}$ rule with $h = \frac{1}{4}$. **03**
- (b) Using Euler's method, find $y(0.1)$ in five steps given that **04**
 $\frac{dy}{dx} = x + y + xy$, $y(0) = 1$.
- (c) Use the Power method to find the largest eigen value and **07**
corresponding eigen vector of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

OR

- (c) Obtain the cubic spline for every subinterval from the given data: **07**

x	0	1	2	3
$f(x)$	1	2	33	244

- Q.3** (a) Find a root of the equation $x = \cos(x)$ by Bisection method correct to two decimal places up to third iteration. **03**
- (b) Solve the given system of equations using Gauss Jacobi method. **04**
 $4x + y + 3z = 17$, $x + 5y + z = 14$, $2x - y + 8z = 12$
- (c) Using Newton's divided difference formula, calculate the value of $f(6)$ from the given data: **07**

x	1	2	7	8
$f(x)$	1	5	5	4

OR

- Q.3** (a) Using appropriate interpolation formula, find the value of $f(1.6)$ if **03**

x	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5

- (b) Using the Trapezoidal rule, evaluate the integral $\int_1^5 \log_{10}(x) dx$ with eight subintervals. 04
- (c) Using fourth order Runge-Kutta method to find the value of y when $x = 0.2$ given that $y' = x + y^2$ and $y = 1$ when $x = 0$. 07
- Q.4** (a) Use the Power method to find the largest eigen value and corresponding eigen vector of the matrix $A = \begin{bmatrix} 4 & 2 \\ 1 & 3 \end{bmatrix}$. Do four steps only. 03
- (b) Use the secant method to find the root of $xe^x - 1 = 0$ correct to three decimal places between 0 and 1. 04
- (c) Solve the given system of equations using Gauss Seidel method. 07
 $6x + y + z = 105$, $4x + 8y + 3z = 155$, $5x + 4y - 10z = 65$
- OR**
- Q.4** (a) Fit a straight line $y = a + bx$ to the following data: 03
- | | | | | | |
|-----|----|----|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 14 | 27 | 40 | 55 | 68 |
- (b) Using Newton's forward interpolation formula, find the value of $f(1.3)$ if 04
- | | | | | |
|--------|-----|-----|-----|------|
| x | 1 | 2 | 3 | 4 |
| $f(x)$ | 1.1 | 4.2 | 9.3 | 16.4 |
- (c) Evaluate $\int_0^1 e^{-x^2} dx$ by using the Gaussian integration with $n = 3$. 07
- Q.5** (a) Using Newton's backward interpolation formula, find the value of $y(300)$ if 03
- | | | | | | |
|-----|-----|-----|-----|-----|------|
| x | 50 | 100 | 150 | 200 | 250 |
| y | 618 | 724 | 805 | 906 | 1032 |
- (b) Define the absolute error, percentage error and relative error. 04
- (c) Using modified Euler's method, solve $y' = 1 - y$, $y(0) = 0$ at $x = 0.1$ and $x = 0.2$. Also, compare with the exact solution. 07
- OR**
- Q.5** (a) Using Lagrange's interpolation formula, find $y(2)$ to the given data: 03
- | | | | | |
|-----|----|---|---|----|
| x | -1 | 0 | 2 | 3 |
| y | 8 | 3 | 1 | 12 |
- (b) Explain the pitfall of elimination method and techniques for improving them. 04
- (c) Solve the given system of equations using Gauss Jordan elimination method. 07
 $10x + y + z = 12$, $2x + 10y + z = 13$, $x + y + 5z = 7$
