

Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY
Bachelor of Engineering - SEMESTER - IV EXAMINATION - WINTER 2025

Subject Code: 3144202

Date: 01-12-2025

Subject Name: Discrete Mathematics and Graph Theory

Time: 02:30 PM TO 05:00 PM

Total Marks: 70

Instructions

- 1. Attempt all questions.**
- 2. Make suitable assumptions wherever necessary.**
- 3. Figures to the right indicate full marks.**
- 4. Simple and non-programmable scientific calculators are allowed.**

	Marks
Q.1 (a) If $A = \{\alpha, \beta\}$ and $B = \{1, 2, 3\}$, find $A \times B$ and $B \times A$.	03
(b) Write product rule for counting. The chairs of an auditorium are to be labeled with a letter and a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?	04
(c) (i) Define surjective function. Is the function $f(x) = x^2$ from the set of integers to the set of integers surjective? (ii) Among 100 people, at least how many of them were born in the same month?	07
Q.2 (a) Using the statements, R: Mark is rich H: Mark is happy, write the following statements in symbolic form: (a) Mark is poor but happy (b) Mark is rich or unhappy (c) Mark is neither rich nor happy.	03
(b) Show that $P \mapsto (Q \mapsto R)$ is logically equivalent to $(P \wedge Q) \mapsto R$.	04
(c) (1) Determine the truth value of each of the following statements if the domain consists of all integers. (a) $\forall n (n + 1 > n)$ (b) $\exists n (2n = 3n)$ (2) Let $P(x)$ be the statement "x can speak Russian" and let $Q(x)$ be the statement "x knows the computer language C++." Express each of following sentences in terms of $P(x)$, $Q(x)$, quantifiers and logical connectives. The domain for quantifiers consists of all students at your school. (a) There is a student at your school who can speak Russian and who knows C++. (b) There is a student at your school who can speak Russian but who does not know C++. (c) Every student at your school either can speak Russian or knows C++. (d) No student at your school can speak Russian or knows C++.	07

OR

- (c) Determine whether $(p \rightarrow (q \rightarrow r)) \rightarrow ((p \rightarrow q) \rightarrow (p \rightarrow r))$ is a tautology or not using truth table. 07

- Q.3 (a)** Consider the relation R on the set $\{1, 2, 3, 4\}$ defined as $R = \{(1, 1), (1, 2), (1, 4), (2, 1), (2, 2), (3, 3), (4, 1), (4, 4)\}$. Is R reflexive, symmetric and antisymmetric? 03

- (b) Draw the Hasse diagram for the partial ordering $\{(A, B) | A \subseteq B\}$ on the power set $P(S)$ where $S = \{a, b, c\}$. 04

- (c) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$ with the initial conditions $a_0 = 2, a_1 = 5$ and $a_2 = 15$ 07

OR

- (a) Determine whether the partially ordered set $(P(S), \subseteq)$ is a lattice where S is a set. 03

- (b) Define (1) complete lattice (2) modular lattice 04

- (c) Define transitive closure of a relation. Find the transitive closure of the relation $R = \{(1, 2), (2, 3), (3, 3)\}$ on $A = \{1, 2, 3\}$ 07

- Q.4 (a)** Let the permutations of the elements of $\{1, 2, 3, 4, 5\}$ be given by $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 4 & 5 \end{pmatrix}$ and $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 4 \end{pmatrix}$. Find $\alpha\beta$ and $\beta\alpha$. Is $\alpha\beta = \beta\alpha$? 03

- (b) Show that the set of all elements a of a group G such that $a * x = x * a$ for every x in G is a subgroup of G . 04

- (c) Define ring, integral domain and field and give one example of each. 07

OR

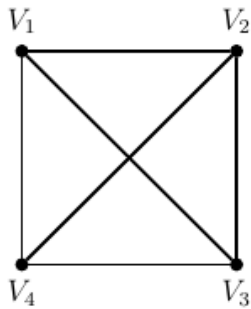
- (a) Let G be the group of real numbers under addition and G' be the group of positive real numbers under multiplication. Show that the mapping $f : G \rightarrow G'$ defined by $f(x) = 2^x$ is a homomorphism. 03

- (b) Consider the set Q of rational numbers and let $*$ be the operation on Q defined by $a * b = a + b - ab$. Find $2 * (-5)$ and $7 * \frac{1}{2}$. Is $(Q, *)$ a semigroup? 04

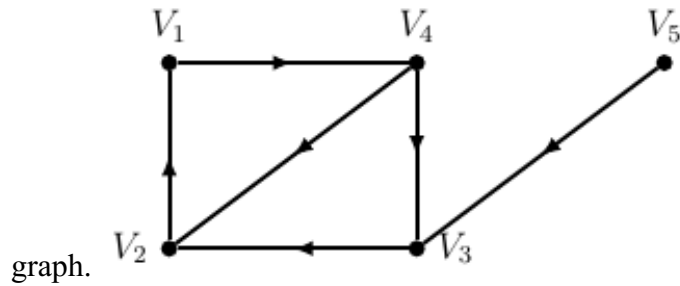
- (c) Define the following. 07
- (1) Weighted graph
 - (2) Node base
 - (3) Converse of a digraph
 - (4) Planar graphs

- Q.5 (a)** Define the following: 03
- (1) Indegree and outdegree of a directed graph
 - (2) Simple path and elementary path

- (b) Determine the number of different paths of length 2 between the vertices V_3 and V_4 in following graph using its adjacency matrix. 04

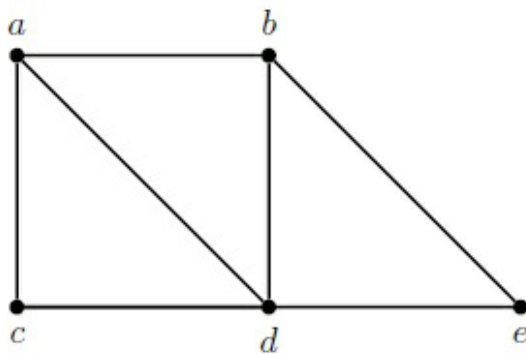


- (c) Use Warshall's algorithm to find path matrix from the adjacency matrix of following 07

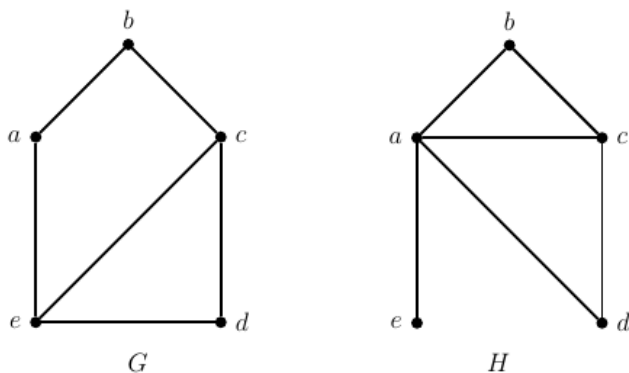


OR

- (a) Define Euler path. Does the following graph have an Euler path? 03



- (b) Determine whether the graphs shown in following figure are isomorphic or not. 04



- (c) When a directed graph is said to be (1) strongly connected and (2) weakly connected? 07
Determine whether the following graph is strongly connected. Is it weakly connected?

