

Enrollment No./Seat No.:

GUJARAT TECHNOLOGICAL UNIVERSITY
Bachelor of Engineering - SEMESTER - IV EXAMINATION - WINTER 2025

Subject Code: 3144803

Date: 24-11-2025

Subject Name: Number Theory

Time: 02:30 PM TO 05:00 PM

Total Marks: 70

Instructions

- 1. Attempt all questions.**
- 2. Make suitable assumptions wherever necessary.**
- 3. Figures to the right indicate full marks.**
- 4. Simple and non-programmable scientific calculators are allowed.**

	Marks
Q.1 (a) Find the GCD of (172,20) using Euclid's Algorithm.	03
(b) What is Mersenne numbers? Explain with an example.	04
(c) Determine all solutions in the positive integers of the following Diophantine equation $172x + 20y = 1000$.	07
Q.2 (a) Solve linear congruence $9x \equiv 21 \pmod{30}$ and find the other incongruent solutions.	03
(b) State Fermat's Theorem and Verify that 17 divides $11^{104} + 1$.	04
(c) Solve the system of linear congruences $x \equiv 1 \pmod{4}$ $x \equiv 2 \pmod{3}$ $x \equiv 3 \pmod{5}$	07

OR

(c) State Chinese Remainder theorem and find the value of x for the given set of congruent equations using Chinese Remainder theorem. $x \equiv 5 \pmod{11}$ $x \equiv 4 \pmod{29}$ $x \equiv 15 \pmod{31}$	07
Q.3 (a) Define Euler's phi-function take any example.	03
(b) State Euler's Theorem and evaluate $2^{100000} \pmod{77}$.	04
(c) Given $n \geq 1$, let $\sigma_s(n)$ denote the sum of the s^{th} powers of the positive divisors of n; that is, $\sigma_s(n) = \sum_{d n} d^s$ Verify the following: (a) $\sigma_0 = T$ and $\sigma_1 = \sigma$. (b) σ_s is a multiplicative function.	07

OR

- (a) The highest power of 3 contained in $31!$ 03
 - (b) Calculate $T(2700)$ and $\sigma(2700)$. 04
 - (c) Write down Mobius inversion formula and find the $\mu(1)$, $\mu(7)$, $\mu(25)$. 07
- Q.4**
- (a) Find all primitive root of 15 03
 - (b) What is Jacobi symbol? State its properties. 04
 - (c) Solve $x^2 \equiv 23 \pmod{7^2}$. 07

OR

- (a) Evaluate Legendre symbol : $(8/11)$ and $(7/13)$. 03
 - (b) Define Euler's solvability criterion with one example. 04
 - (c) Solve $4x^9 \equiv 7 \pmod{13}$. 07
- Q.5**
- (a) Explain Asymmetric key Encryption with Private Key. 03
 - (b) If the Caesar cipher produced "ZCQR MD JS AI" what is the plain text message? 04
 - (c) For $p = 11$ and $q = 19$ and choose $d = 17$. Apply RSA algorithm where Cipher message = 80 and thus find the plain text. 07

OR

- (a) Explain Asymmetric key Encryption with Public Key. 03
- (b) Explain Knapsack Cryptosystem. 04
- (c) P and Q are two prime numbers. $P = 7$, and $Q = 17$. Take public key $E = 5$. If the plain text value is 10, then what will be the cipher text value according to the RSA algorithm? 07
