Seat No.: _____ Enrolment No._____

GUJARAT TECHNOLOGICAL UNIVERSITY

DIPLOMA ENGINEERING – SEMESTER-II EXAMINATION – Summer- 2019

Subject Code: 3320002 Date: 03-06-2019

Subject Name: ADVANCED MATHEMATICS (GROUP-1)

Time: 10:30 AM to 01:00 PM Total Marks: 70

Instructions:

1. **Attempt all questions.**
2. **Make Suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Use of programmable & Communication aids are strictly prohibited.**
5. **Use of only simple calculator is permitted in Mathematics.**
6. **English version is authentic.**

Q.1 Fill in the blanks using appropriate choice from the given options. **14**
(યોગ્ય વિકલ્પ પસંદ કરી ખાલી જગ્યા પૂરો.)

1 If $3x + 2yi = 6 + 4i$ then $x =$ _____, $y =$ _____
(જો $3x + 2yi = 6 + 4i$ હોય તો $x =$ _____, $y =$ _____)
(a) 2, 2 (b) 2, 3 (c) 3, 2 (d) 1, 1

2 $i + i^2 + i^3 + i^4 =$ _____
(a) -1 (b) 0 (c) 1 (d) i

3 If $z = (3 - 4i)$ then $|z| =$ _____
(જો $z = (3 - 4i)$ હોય તો $|z| =$ _____)
(a) -5 (b) 25 (c) 5 (d) 0

4 If f(x) = $x^3 + 5$ then f(2) = _____
(a) 13 (b) 5 (c) 0 (d) -3

5 If f(x) = $\log x$ then f(x) + f(y) = _____
(જો f(x) = $\log x$ હોય તો f(x) + f(y) = _____)
(a) $f(x + y)$ (b) $f(x - y)$ (c) $f(x \cdot y)$ (d) $f\left(\frac{x}{y}\right)$

6 $\lim_{x \to 1}(x^3 - 3x^2 + 5x - 6) =$ _____
(a) 1 (b) 6 (c) 3 (d) -3

7 $\lim_{x \to 2} \frac{x^3 - 8}{x - 2} =$ _____
(a) -12 (b) 0 (c) 8 (d) 12

8 $\lim_{\theta \to 0} \frac{tan\theta}{\theta} =$ _____
(a) -1 (b) 0 (c) 1 (d) ∞

9 $\frac{d}{dx}(secx)$=_____
(a) $sec^2 x$ (b) $-sec^2 x$ (c) $secx \cdot tanx$ (d) $-secx \cdot tanx$

10 $\frac{d}{dx}(sin^2 x + cos^2 x)$=_____
(a) 1 (b) -1 (c) 0 (d) none of these

11 $\int x^7 dx$ = _____+ c
(a) $\frac{x^8}{8}$ (b) $\frac{x^6}{6}$ (c) $7x^6$ (d) $7logx$

12 $\int_{-2}^{2} x^3 dx$=_____
(a) -1 (b) 0 (c) 1 (d) $\frac{1}{4}$

13 The order of the differential equation $\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right)^3 + 4y = 0$ is _____

(વિકલ સમીકરણ $\frac{d^2y}{dx^2} - 3\left(\frac{dy}{dx}\right)^3 + 4y = 0$ ની કક્ષા _____ થશે.)

(a) 1 (b) 0 (c) 2 (d) 3

14 The degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + 5\left(\frac{dy}{dx}\right)^2 - y = 0$ is _____

(વિકલ સમીકરણ $\left(\frac{d^2y}{dx^2}\right)^3 + 5\left(\frac{dy}{dx}\right)^2 - y = 0$ નું પરિમાણ _____ થશે.)

(a) 1 (b) 3 (c) 0 (d) 2

Q.2 **(a)** Attempt any two. (કોઈપણ બે ગણો.) **06**

1. Simplify (સાદુરૂપ આપો):
$$\frac{(cos3\theta + isin3\theta)^{-4}(cos\theta - isin\theta)^5}{(cos2\theta - isin2\theta)^6(cos13\theta + isin13\theta)}$$
2. If $z = \frac{3+7i}{1-i}$ then find its conjugate complex number and modulus .
(જો $z = \frac{3+7i}{1-i}$ હોય તો તેની અનુબદ્ધ સંકર સંખ્યા અને માનાંક શોધો)
3. Find the value (કિંમત શોધો): $\lim_{x\to 0} \frac{4^x - 3^x}{x}$

(b) Attempt any two. (કોઈપણ બે ગણો.) **08**

1. Find square root of $3 + 4\sqrt{10}i$
$(3 + 4\sqrt{10}i$ નું વર્ગમૂળ શોધો).
2. Convert $1 - \sqrt{3}i$ into polar form.
$(1 - \sqrt{3}i$ ને ધ્રુવીય સ્વરૂપમાં ફેરવો.)
3. If f(x) = $\log\left(\frac{x-1}{x}\right)$ then prove that f(x) + f(-x) = f(x^2).
(જો f(x) = $\log\left(\frac{x-1}{x}\right)$ હોય તો સાબિત કરો કે f(x) + f(-x) = f(x^2).)

Q.3 **(a)** Attempt any two. (કોઈપણ બે ગણો.) **06**

1. $\lim_{x\to -1} \frac{2x^3 + 5x^2 + 4x + 1}{3x^3 + 5x^2 + x - 1}$
2. $\lim_{n\to\infty} \frac{3n^3 - 4n^2 - n - 5}{2n^3 + 3n^2 - 2n + 7}$
3. (i) $\lim_{\theta\to 0} \frac{\sin 2\theta}{\tan 7\theta}$ (ii) $\lim_{x\to\infty} \left(1 + \frac{2}{x}\right)^{2x}$

(b) Attempt any two. (કોઈપણ બે ગણો.) **08**

1. Solve the differential equation (વિકલ સમીકરણ ઉકેલો)
$x(1 + y^2)dx = y(1 + x^2)dy$
2. Solve the differential equation (વિકલ સમીકરણ ઉકેલો)
$tanydx + tanx \cdot sec^2 ydy = 0$
3. Solve the differential equation (વિકલ સમીકરણ ઉકેલો)
$\frac{dy}{dx} + 2y = e^x$

Q.4 **(a)** Attempt any two. (કોઈપણ બે ગણો.) **06**

1. If $x = e^t + sint, y = logt + cost$ then find $\frac{dy}{dx}$.
(જો $x = e^t + sint, y = logt + cost$ હોય તો $\frac{dy}{dx}$ શોધો.)
2. Find velocity (v) and acceleration (a) at t = 2 for the equation of motion $s = 2t^3 + 3t^2 - 12t + 5$.
(ગતિસૂત્ર $s = 2t^3 + 3t^2 - 12t + 5$ હોય તો વેગ (v) અને પ્રવેગ (a), t = 2 આગળ શોધો.)

3. If y = Acospt + Bsinpt then prove that $y''+p^2y = 0$.
(જો y = Acospt + Bsinpt હોય તો સાબિત કરો કે $y''+p^2y = 0$.)

(b) Attempt any two. (કોઈપણ બે ગણો.) **08**

1. Differentiate $y = e^x$ using the defintion.
($y = e^x$ નું વ્યાખ્યાની મદદથી વિકલન શોધો.)
2. If $y = (\sin x)^x$ then find y'. (જો $y = (\sin x)^x$ હોય તો y' શોધો.)
3. Find the maximum and minimum value of $f(x) = 2x^3 - 15x^2 + 36x + 10$.
($f(x) = 2x^3 - 15x^2 + 36x + 10$ માટે મહત્તમ અને ન્યુનતમ કિંમત શોધો.)

Q.5 **(a)** Attempt any two. (કોઈપણ બે ગણો.) **06**

1. $\int \frac{x-5}{(x-1)(x-2)}\,dx$
2. $\int_{-1}^{1} \frac{x^3-27}{x-3}\,dx$
3. $\int \cos 5x \cdot \sin 2x\,dx$

(b) Attempt any two. (કોઈપણ બે ગણો.) **08**

1. $\int_{0}^{1} \frac{dx}{x+\sqrt{1-x^2}}$
2. $\int e^x x^3\,dx$
3. Find the area bounded by the curve $ay = x^2$, line x = a and X-axis.
(વક્રો $ay = x^2$, રેખા x = a અને X-અક્ષ વચ્ચે ઘેરાયેલા પ્રદેશનું ક્ષેત્રફળ શોધો.)