

Seat No. / Enrolment No.:

GUJARAT TECHNOLOGICAL UNIVERSITY

Diploma Engineering – SEMESTER – 1 (OLD) – EXAMINATION – Summer-2024

Subject Code: 3300001

Date: 06-06-2024

Subject Name: Basic Mathematics

Time: 10:30 AM TO 01:00 PM

Total Marks: 70

Instructions:

1. Attempt all questions.
2. Make Suitable assumptions wherever necessary.
3. Figures to the right indicate full marks.
4. Use of programmable & Communication aids are strictly prohibited.
5. Use of non-programmable scientific calculator is permitted.
6. English version is authentic.

Q-1 Fill in the blanks using appropriate choice from the given options.

(14)

(1) If $\log_7 x = 1$ then $x =$ _____
(a) 1 (b) 0 (c) 7 (d) -7

(1) જો $\log_7 x = 1$ તો $x =$ _____
(a) 1 (b) 0 (c) 7 (d) -7

(2) $\log_5(125) =$ _____
(a) 5 (b) 3 (c) 2 (d) -25

(2) $\log_5(125) =$ _____
(a) 5 (b) 3 (c) 2 (d) -25

(3) $a^{\log_a b} =$ _____
(a) 0 (b) a (c) b (d) 1

(3) $a^{\log_a b} =$ _____
(a) 0 (b) a (c) b (d) 1

(4) $\begin{vmatrix} \log_6 3 & -1 \\ \log_6 2 & 1 \end{vmatrix} =$ _____
(a) 0 (b) -1 (c) 1 (d) 2

(4) $\begin{vmatrix} \log_6 3 & -1 \\ \log_6 2 & 1 \end{vmatrix} =$ _____
(a) 0 (b) -1 (c) 1 (d) 2

(5) If $A = \begin{bmatrix} d & b \\ c & d \end{bmatrix}$ then $\text{adj } A =$ _____
(a) $\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$ (b) $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ (c) $\begin{bmatrix} d & -b \\ -c & d \end{bmatrix}$ (d) $\begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$

(5) જો $A = \begin{bmatrix} d & b \\ c & d \end{bmatrix}$ તો $\text{adj } A =$ _____
(a) $\begin{bmatrix} a & -b \\ -c & d \end{bmatrix}$ (b) $\begin{bmatrix} a & c \\ b & d \end{bmatrix}$ (c) $\begin{bmatrix} d & -b \\ -c & d \end{bmatrix}$ (d) $\begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$

(6) If $\begin{bmatrix} 0 & x & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = [4]$ then $x =$ _____
(a) 2 (b) 4 (c) 5 (d) 6

(6) જો $\begin{bmatrix} 0 & x & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = [4]$ તો $x =$ _____

(a) 2 (b) 4 (c) 5 (d) 6

(7) Order of matrix $\begin{bmatrix} 2 & 1 & 5 \\ 6 & 0 & 1 \end{bmatrix}$ is = _____

(a) 3×3 (b) 3×2 (c) 2×3 (d) none of these

(7) શ્રેણિક $\begin{bmatrix} 2 & 1 & 5 \\ 6 & 0 & 1 \end{bmatrix}$ ની કક્ષા _____ છે.

(a) 3×3 (b) 3×2 (c) 2×3 (d) none of these

(8) $\tan^{-1} x + \cot^{-1} x =$ _____

(a) 1 (b) 0 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

(8) $\tan^{-1} x + \cot^{-1} x =$ _____

(a) 1 (b) 0 (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{6}$

(9) Period of $\sin(2x + 3)$ is _____

(a) π (b) $-\pi$ (c) 4π (d) $\frac{\pi}{2}$

(9) $\sin(2x + 3)$ નું અવર્તમાન _____ છે.

(a) π (b) $-\pi$ (c) 4π (d) $\frac{\pi}{2}$

(10) $\sin(135^\circ) =$ _____

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

(10) $\sin(135^\circ) =$ _____

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}$ (d) $-\sqrt{2}$

(11) $\frac{4\pi}{9}$ radian = _____

(a) 40° (b) 80° (c) 20° (d) 10°

(11) $\frac{4\pi}{9}$ રેડિયન = _____

(a) 40° (b) 80° (c) 20° (d) 10°

(12) Volume of sphere of radius 'r' is _____

(a) $2\pi r^3$ (b) $\frac{4}{3}\pi r^3$ (c) $4\pi r^2$ (d) $\frac{2}{3}\pi r^3$

(12) 'r' ત્રિજ્યા વાળા ગોળનું ઘનફળ _____ છે.

(a) $2\pi r^3$ (b) $\frac{4}{3}\pi r^3$ (c) $4\pi r^2$ (d) $\frac{2}{3}\pi r^3$

(13) If perimeter of square is 40cm , then area is _____

(a) 1600 cm^2 (b) 20 cm^2 (c) 10 cm^2 (d) 100 cm^2

(13) જો ચોરસ ની પરિમિતિ 40 સેમી છે તો ક્ષેત્રફળ _____ છે.

(a) 1600 cm^2 (b) 20 cm^2 (c) 10 cm^2 (d) 100 cm^2

(14) Formula for closed surface area of cylinder is _____

(a) $\pi r(h + r)$ (b) $2\pi rh$ (c) $2\pi r(h + r)$ (d) $\pi r^2 h$

(14) બંધ નળકાર નું પૃષ્ઠફળ _____ છે.

(a) $\pi r(h + r)$ (b) $2\pi rh$ (c) $2\pi r(h + r)$ (d) $\pi r^2 h$

Q-2 (A) Attempt Any Two (કોઈ પણ બે લખો)**(6)**

(1) Solve : $\log(2x + 1) + \log(3x - 1) = 0$

(2) If Curve surface area of sphere is 616sq. cm then find diameter of sphere.

(2) જો ગોલક ની વક્રસપાટી નું ક્ષેત્રફળ 616 ચો. સેમી છે તો ગોલક નો વ્યાસ શોધો.

(3) For triangle ΔABC , measurement of sides $a = 8\text{cm}$, $b = 10\text{cm}$ and $c = 6\text{cm}$ then find area of triangle.(3) ΔABC માટે, બાજુ ઓ નીમાપ $a = 8$ સેમી, $b = 10$ સેમી and $c = 6$ સેમી તો ત્રિકોણ નું ક્ષેત્રફળ શોધો.**(B) Attempt Any Two (કોઈ પણ બે લખો)****(8)**

(1) Prove that $\log(x + \sqrt{x^2 - 1}) + \log(x - \sqrt{x^2 - 1}) = 0$

(1) સાબિત કરો કે $\log(x + \sqrt{x^2 - 1}) + \log(x - \sqrt{x^2 - 1}) = 0$

(2) If $\log\left(\frac{a+b}{2}\right) = \frac{1}{2} [\log a + \log b]$ then prove that $a = b$

(2) જો $\log\left(\frac{a+b}{2}\right) = \frac{1}{2} [\log a + \log b]$ સાબિત કરો કે $a = b$

(3) How many spherical balls of radius 2cm can be made from cube of length 44 cm.

(3) 44 સેમી લંબાઈ વાળા સમઘન માથી 2સેમી ત્રિજ્યાવાળા કેટલા દડા બનાવી શકાય.

Q-3 (A) Attempt Any Two (કોઈ પણ બે લખો)**(6)**

(1) If $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$ then find $3A - 2B$.

(1) જો $A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \\ 2 & 5 \end{bmatrix}$ અને $B = \begin{bmatrix} -1 & -2 \\ 0 & 5 \\ 3 & 1 \end{bmatrix}$ તો $3A - 2B$ શોધો.

(2) If $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$ then find AB and BA

(2) જો $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ અને $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 1 & 2 \end{bmatrix}$ AB અને BA શોધો.

(3) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ then prove that $A^2 - 5A + 7I = 0$.

(3) જો $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ સાબિત કરો કે $A^2 - 5A + 7I = 0$.

(B) Attempt Any Two (કોઈ પણ બે લખો)**(8)**

(1) $A = \begin{bmatrix} -1 & 2 & 3 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix}$ then find AB . If AB is possible ? Given reason.

(1) $A = \begin{bmatrix} -1 & 2 & 3 \\ 3 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ અને $B = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -1 & 5 \end{bmatrix}$ તો AB શોધો. જો AB શક્ય હોય તો ? કારણ આપો.

(2) If $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ then find A^{-1}

(2) જો $A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$ A^{-1} શોધો.

(3) Solve Equation $3x - 2y = 8$ and $5x + 4y = 6$ using matrix method.

(3) શ્રેણિક ની મદદ થી ઉકેલો : $3x - 2y = 8$ અને $5x + 4y = 6$

Q-4 (A) Attempt Any Two (કોઈ પણ બે લખો)

(6)

(1) Prove that $\frac{\sin(\theta - \frac{\pi}{2})}{\cos(\theta - \pi)} + \frac{\tan(\frac{\pi}{2} - \theta)}{\cot(\pi + \theta)} + \frac{\operatorname{cosec}(\frac{\pi}{2} + \theta)}{\sec(\pi + \theta)} = 1$

(1) સાબિત કરો કે : $\frac{\sin(\theta - \frac{\pi}{2})}{\cos(\theta - \pi)} + \frac{\tan(\frac{\pi}{2} - \theta)}{\cot(\pi + \theta)} + \frac{\operatorname{cosec}(\frac{\pi}{2} + \theta)}{\sec(\pi + \theta)} = 1$

(2) Prove that $\tan 20^\circ + \tan 25^\circ + \tan 20^\circ \tan 25^\circ = 1$

(2) સાબિત કરો કે : $\tan 20^\circ + \tan 25^\circ + \tan 20^\circ \tan 25^\circ = 1$

(3) Prove that : $\frac{\sin \theta + \sin 2\theta + \sin 4\theta + \sin 5\theta}{\cos \theta + \cos 2\theta + \cos 4\theta + \cos 5\theta} = \tan 3\theta$

(3) સાબિત કરો કે : $\frac{\sin \theta + \sin 2\theta + \sin 4\theta + \sin 5\theta}{\cos \theta + \cos 2\theta + \cos 4\theta + \cos 5\theta} = \tan 3\theta$

(B) Attempt Any Two (કોઈ પણ બે લખો)

(8)

(1) Draw the graph of $y = \cos x$, where $0 \leq x \leq \pi$

(1) $y = \cos x$, where $0 \leq x \leq \pi$ નો આલેખ દોરો.

(2) Prove that $\tan^{-1}\left(\frac{5}{7}\right) + \tan^{-1}\left(\frac{1}{6}\right) = \frac{\pi}{4}$

(2) સાબિત કરો કે $\tan^{-1}\left(\frac{5}{7}\right) + \tan^{-1}\left(\frac{1}{6}\right) = \frac{\pi}{4}$

(3) Prove that $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan\left(\frac{\theta}{2}\right)$

(3) સાબિત કરો કે $\frac{1 + \sin \theta - \cos \theta}{1 + \sin \theta + \cos \theta} = \tan\left(\frac{\theta}{2}\right)$

Q-5 (A) Attempt Any Two (કોઈ પણ બે લખો)

(6)

(1) If $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ and $\vec{c} = (-1, 2, -5)$ then find $|\vec{a} + 2\vec{b} - \vec{c}|$

(1) જો $\vec{a} = (3, -1, -4)$, $\vec{b} = (-2, 4, -3)$ અને $\vec{c} = (-1, 2, -5)$ તો $|\vec{a} + 2\vec{b} - \vec{c}|$ શોધો..

(2) Find the unit vector perpendicular to both the vectors $\vec{a} = (3, -1, 2)$ and $\vec{b} = (2, 1, -1)$

(2) સદીશો $\vec{a} = (3, -1, 2)$ અને $\vec{b} = (2, 1, -1)$ માટે લંબ એકમ સદીશ શોધો.

(3) What value of p vectors $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$ and $\vec{b} = p\vec{i} - \vec{j} + 3\vec{k}$ are perpendicular.

(3) p ના કયાં મૂલ્ય માટે સદીશો $\vec{a} = 2\vec{i} + 3\vec{j} - \vec{k}$ અને $\vec{b} = p\vec{i} - \vec{j} + 3\vec{k}$ પરસ્પર લંબ હોય.

(B) Attempt Any Two (કોઈ પણ બે લખો)

(8)

(1) If angle between the vectors $\vec{i} + \vec{j} - \vec{k}$ and $2\vec{i} - 2\vec{j} + \vec{k}$ is θ , then prove that

$$\theta = \sin^{-1}\left(\sqrt{\frac{26}{27}}\right).$$

(1) સદીશો $\vec{i} + \vec{j} - \vec{k}$ અને $2\vec{i} - 2\vec{j} + \vec{k}$ નો ખૂણો θ , તો સાબિત કરો કે $\theta = \sin^{-1}\left(\sqrt{\frac{26}{27}}\right)$

(2) Force $(1, 2, 3)$ and $(3, 1, 1)$ are acting on a particle and the particle moves from $(0, 1, -2)$ to $(5, 1, 2)$. Find the work done.

(2) એક કણ પર અચળ બળો $(1, 2, 3)$ અને $(3, 1, 1)$ લાગે છે. આ બળોની અસર હેઠળ કણ, બિંદુ $(0, 1, -2)$ થી બિંદુ $(5, 1, 2)$ ખસે છે તો થયેલ કાર્ય શોધો.

(3) If $\vec{a} = 3\vec{i} - \vec{j} - 4\vec{k}$, $\vec{b} = -2\vec{i} + 4\vec{j} - 3\vec{k}$ and $\vec{c} = \vec{i} + 2\vec{j} - \vec{k}$ then find direction cosine of $3\vec{a} - 2\vec{b} + 4\vec{c}$

(3) જો $\vec{a} = 3\vec{i} - \vec{j} - 4\vec{k}$, $\vec{b} = -2\vec{i} + 4\vec{j} - 3\vec{k}$ અને $\vec{c} = \vec{i} + 2\vec{j} - \vec{k}$ તો $3\vec{a} - 2\vec{b} + 4\vec{c}$ નો દિશાકોસાઇન શોધો.