

Seat No.: _____

Enrolment No. _____

GUJARAT TECHNOLOGICAL UNIVERSITY

DIPLOMA ENGINEERING (NEW)– SEMESTER –2 (NEW) EXAMINATION – WINTER-2020

Subject Code: 3320002

Date: 17-03-2021

Subject Name: Advanced Mathematics

Time: 02:30 PM TO 04:30 PM

Total Marks: 56

Instructions:

1. Attempt any FOUR Questions from Q.1 to Q.5.
2. Attempt ALL questions.
3. Make Suitable assumptions wherever necessary.
4. Figures to the right indicate full marks.
5. Use of SIMPLE CALCULATOR is permissible.(Scientific/Higher Version not allowed)
6. English version is authentic.

Q.1 Fill in the blanks using appropriate choice from the given options.

(14)

યોગ્ય વિકલ્પ પસંદ કરી ખાલી જગ્યા પૂરો.

(1) If $z = 3 + 2i$ then $\bar{z} =$ _____.

- (a) $2 + 3i$ (b) $3 - 2i$ (c) $-2 - 3i$ (d) $2 - 3i$

(૧) જો $z = 3 + 2i$ હોય તો $\bar{z} =$ _____.

- (a) $2 + 3i$ (b) $3 - 2i$ (c) $-2 - 3i$ (d) $2 - 3i$

(2) $i^{11} =$ _____.

- (a) -1 (b) 1 (c) i (d) $-i$

(૨) $i^{11} =$ _____.

- (a) -1 (b) 1 (c) i (d) $-i$

(3) If $z = 3 - 4i$ then $|z| =$ _____.

- (a) 7 (b) 25 (c) 5 (d) 1

(૩) જો $z = 3 - 4i$ હોય તો $|z| =$ _____.

- (a) 7 (b) 25 (c) 5 (d) 1

(4) $z + \bar{z} =$ _____.

- (a) $2Im(z)$ (b) $2Re(z)$ (c) $Im(z)$ (d) $Re(z)$

(૪) $z + \bar{z} =$ _____.

- (a) $2Im(z)$ (b) $2Re(z)$ (c) $Im(z)$ (d) $Re(z)$

(5) $\lim_{\theta \rightarrow 0} \frac{\tan 3\theta}{4\theta} =$ _____.

- (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 12 (d) $\frac{1}{12}$

(૫) $\lim_{\theta \rightarrow 0} \frac{\tan 3\theta}{4\theta} =$ _____.

- (a) $\frac{3}{4}$ (b) $\frac{4}{3}$ (c) 12 (d) $\frac{1}{12}$
- (6) $\lim_{x \rightarrow 2} \frac{x^2 - 1}{x - 1} = \underline{\hspace{2cm}}$.
- (a) 3 (b) $\frac{1}{3}$ (c) 2 (d) 1
- (7) $\lim_{x \rightarrow 2} \frac{x^2 - 1}{x - 1} = \underline{\hspace{2cm}}$.
- (a) 3 (b) $\frac{1}{3}$ (c) 2 (d) 1
- (8) $\frac{d}{dx}(\sin^{-1} x + \cos^{-1} x) = \underline{\hspace{2cm}}$.
- (a) 1 (b) 0 (c) $\frac{\pi}{2}$ (d) π
- (9) $\frac{d}{dx}(\sin^{-1} x + \cos^{-1} x) = \underline{\hspace{2cm}}$.
- (a) 1 (b) 0 (c) $\frac{\pi}{2}$ (d) π
- (10) If $y = x^2$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.
- (a) x (b) 2 (c) $2x$ (d) $\frac{x^3}{3}$
- (11) If $y = x^2$ then $\frac{dy}{dx} = \underline{\hspace{2cm}}$.
- (a) x (b) 2 (c) $2x$ (d) $\frac{x^3}{3}$
- (12) $\frac{d}{dx}(x \ln x) = \underline{\hspace{2cm}}$.
- (a) x (b) $\ln x$ (c) $1 + \ln x$ (d) $1 + x$
- (13) $\frac{d}{dx}(x \ln x) = \underline{\hspace{2cm}}$.
- (a) x (b) $\ln x$ (c) $1 + \ln x$ (d) $1 + x$
- (14) If $x^2 + y^2 = 25$ then $\frac{dy}{dx}$ at (1,2) = $\underline{\hspace{2cm}}$.
- (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) -2
- (15) If $x^2 + y^2 = 25$ then $\frac{dy}{dx}$ at (1,2) = $\underline{\hspace{2cm}}$.
- (a) $-\frac{1}{2}$ (b) $\frac{1}{2}$ (c) 2 (d) -2
- (16) $\int x^2 dx = \underline{\hspace{2cm}} + c$.
- (a) $2x$ (b) $\frac{x^3}{3}$ (c) x (d) 2
- (17) $\int x^2 dx = \underline{\hspace{2cm}} + c$.
- (a) $2x$ (b) $\frac{x^3}{3}$ (c) x (d) 2
- (18) $\int_{-1}^1 x^3 dx = \underline{\hspace{2cm}}$.
- (a) 0 (b) 2 (c) $\frac{1}{4}$ (d) $\frac{x^4}{4}$

(૧૨) $\int_{-1}^1 x^3 dx = \underline{\hspace{2cm}}$.

- (a) 0 (b) 2 (c) $\frac{1}{4}$ (d) $\frac{x^4}{4}$

(13) The integrating factor of $\frac{dy}{dx} + y = 7$ is $\underline{\hspace{2cm}}$.

- (a) e^x (b) x (c) e (d) 1

(૧૩) $\frac{dy}{dx} + y = 7$ ને સંકલનકારક અવયવ $\underline{\hspace{2cm}}$ છે.

- (a) e^x (b) x (c) e (d) 1

(14) The order of $x^3 \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^4 + y = 7$ is $\underline{\hspace{2cm}}$.

- (a) 1 (b) 2 (c) 3 (d) 4

(૧૪) $x^3 \frac{d^2y}{dx^2} + x \left(\frac{dy}{dx}\right)^4 + y = 7$ ને ક્રમ $\underline{\hspace{2cm}}$ છે.

- (a) 1 (b) 2 (c) 3 (d) 4

Q.2 (A) Attempt any two. કોઈપણ બે ગણો.

06

(1) Express $\frac{2+3i}{1-i}$ into $a + bi$ form.

(૧) $\frac{2+3i}{1-i}$ ને $a + bi$ સ્વરૂપમાં ફેરવો.

(2) Express $\frac{\cos 5\theta + i \sin 5\theta}{\cos 2\theta - i \sin 2\theta}$ into $a + bi$ form.

(૨) $\frac{\cos 5\theta + i \sin 5\theta}{\cos 2\theta - i \sin 2\theta}$ ને $a + bi$ સ્વરૂપમાં ફેરવો.

(3) Express $3 + 2i$ in polar form.

(૩) $3 + 2i$ ને ધ્રુવીય સ્વરૂપમાં ફેરવો.

(B) Attempt any two. કોઈપણ બે ગણો.

08

(1) If $f(x) = \log x$ then prove that (i) $f(x) + f(y) = f(xy)$

(ii) $f(x) - f(y) = f\left(\frac{x}{y}\right)$.

(૧) જો $f(x) = \log x$ હોય તો સાબિત કરો કે (i) $f(x) + f(y) = f(xy)$

(ii) $f(x) - f(y) = f\left(\frac{x}{y}\right)$.

(2) Evaluate : $\lim_{n \rightarrow \infty} \frac{\sum n^2}{n^3}$

(૨) ઉકેલો : $\lim_{n \rightarrow \infty} \frac{\sum n^2}{n^3}$

(3) If $f(x) = \frac{1-x}{1+x}$ then prove that $x = f(y)$.

(૩) જો $f(x) = \frac{1-x}{1+x}$ હોય તો સાબિત કરો કે $x = f(y)$.

Q.3 (A) Attempt any two. કોઈપણ બે ગણો.

06

(1) Find $\frac{d}{dx}(x^2)$ using definition.

(૧) વ્યાખ્યાની મદદથી $\frac{d}{dx}(x^2)$ મેળવો.

(2) If $x = \cos \theta$ & $y = \sin \theta$ then find $\frac{dy}{dx}$.

(૨) જો $x = \cos \theta$ તથા $y = \sin \theta$ હોય તો $\frac{dy}{dx}$ મેળવો.

(3) If $y = \sin(e^{x^2+1})$ then find $\frac{dy}{dx}$.

(૩) જો $y = \sin(e^{x^2+1})$ હોય તો $\frac{dy}{dx}$ મેળવો.

(B) Attempt any two. કોઈપણ બે ગણો.

08

(1) If $y = \sin x^x$ then find $\frac{dy}{dx}$.

(1) જો $y = \sin x^x$ હોય તો $\frac{dy}{dx}$ મેળવો.

(2) Find maxima & minima for $f(x) = x^3 - 3x + 11$.

(2) $f(x) = x^3 - 3x + 11$ માટે મહત્તમ તથા ન્યૂનતમ કિંમત મેળવો.

(3) The equation of motion of particle is $s = t^3 - 5t + 7$.

Find velocity & acceleration at $t = 2$.

(૩) એક કણની ગતિનું સમીકરણ $s = t^3 - 5t + 7$ છે. $t = 2$ માટે વેગ તથા પ્રવેગ મેળવો.

Q.4 (A) Attempt any two. કોઈપણ બે ગણો.

06

(1) Evaluate: $\int \left(\frac{x^3+5x^2-2x+7}{x^2} \right) dx$

(૧) ઉકેલો: $\int \left(\frac{x^3+5x^2-2x+7}{x^2} \right) dx$

(2) Evaluate : $\int_0^1 x e^x dx$

(૨) ઉકેલો: $\int_0^1 x e^x dx$

(3) Evaluate : $\int \frac{2x+3}{x^2+3x-2} dx$

(૩) ઉકેલો: $\int \frac{2x+3}{x^2+3x-2} dx$

(B) Attempt any two. કોઈપણ બે ગણો.

08

(1) Evaluate : $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$

(૧) ઉકેલો: $\int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$

(2) Find the area bounded by the line $y = x, x = 0, x = 2$ & x axis.

(૨) રેખા $y = x, x = 0, x = 2$ તથા x અક્ષ વડે ઘેરાયેલા પ્રદેશનું ક્ષેત્રફળ મેળવો.

(3) Evaluate : $\int \frac{\tan^{-1} x}{1+x^2} dx$

(૩) ઉકેલો: $\int \frac{\tan^{-1} x}{1+x^2} dx$

Q.5 (A) Attempt any two. કોઈપણ બે ગણો.

06

(1) Evaluate : $\lim_{x \rightarrow 0} \frac{e^x - \sin x - 1}{x}$

(૧) ઉકેલો: $\lim_{x \rightarrow 0} \frac{e^x - \sin x - 1}{x}$

(2) Form a differential equation using $y = 4 \sin x + 3 \cos x$

(૨) $y = 4 \sin x + 3 \cos x$ માટે વિકલ સમીકરણ મેળવો.

(3) Solve : $\frac{dy}{dx} = \frac{2x}{1+x^2}$

(૩) ઉકેલો: $\frac{dy}{dx} = \frac{2x}{1+x^2}$

(B) Attempt any two. કોઈપણ બે ગણો.

08

(1) Solve : $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$

(૧) ઉકેલો: $\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2}$

(2) Solve : $\frac{dy}{dx} + \frac{y}{x} = 5$

(૨) ઉકેલો: $\frac{dy}{dx} + \frac{y}{x} = 5$

(3) Solve : $2 \frac{dy}{dx} = \frac{y(x+2)}{x}$

(૩) ઉકેલો: $2 \frac{dy}{dx} = \frac{y(x+2)}{x}$